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Lecture ⑥: "pb solving 1.4.7 p. 33 & Bivariate random Variable"

* pb 1.4.7 p. 33

Given independent exponentially distributed random variables S and T with common parameter λ, determine the probability density function of the sum $R = S + T$ and identify its type by name.

Remember

$$f_X(x) = \frac{\lambda^2}{\Gamma(n)} x^{n-1} e^{-\lambda x}, \quad x \geq 0$$

Ans: $\because S, T \sim exp(\lambda)$

$\because R = S + T$

$\therefore R \sim Gamma(2, \lambda)$ ($n = 2$)

$$\therefore f_R(r) = \frac{\lambda^2}{\Gamma(2)} r^{2-1} e^{-\lambda r}, \quad r \geq 0$$

$X \to R$

$x \to r$

$$\therefore f_R(r) = \frac{\lambda^2}{1!} r e^{-\lambda r}$$

$$\therefore f_R(r) = \lambda^2 r e^{-\lambda r}, \quad r \geq 0$$

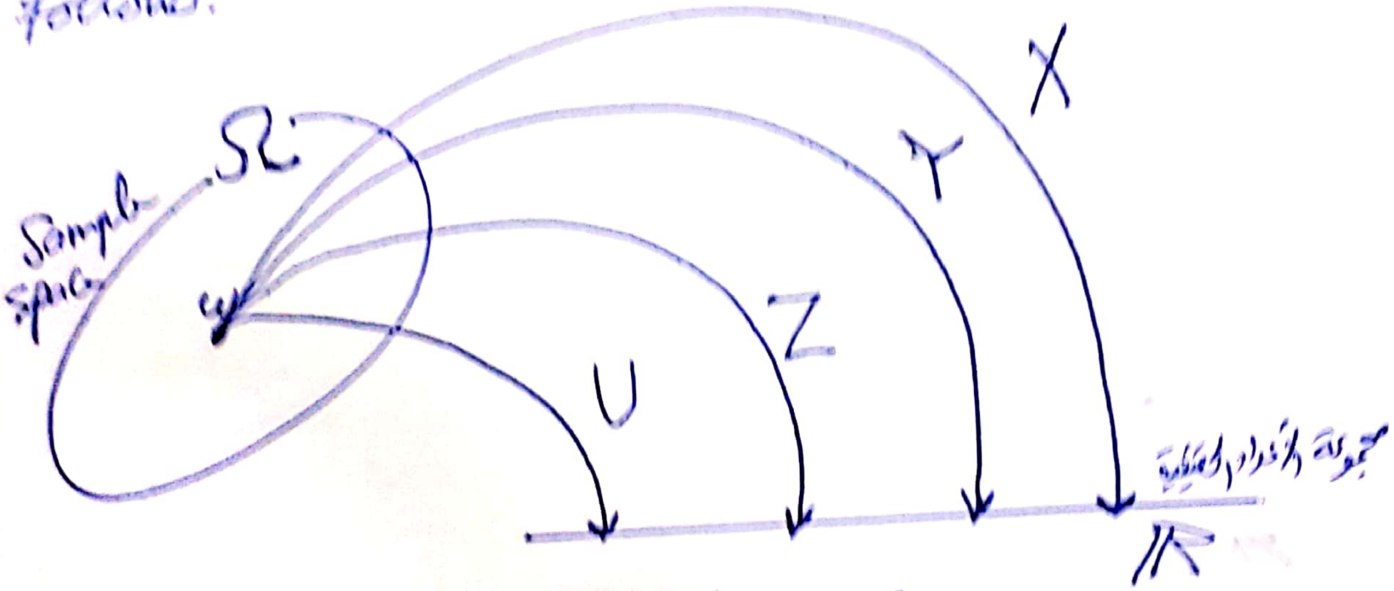
which is the Gamma prob. density fn.

Bivariate random Variable

Random experiment:

* Example

Consider a selection of a student ω in KSU as follows.



when X, Y, Z and U are r.v's represent the age, weight, height and family members for the student then we say that

(X, Y, Z, U) is a multivariate r.v

(X, Y) is a bivariate r.v

and X is a univariate r.v.

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Discrete Bivariate distn

The joint prob. mass fn for discrete Bivariate r.v (X,Y) is

$$P_{X,Y}(x,y) = pr(X=x,\ Y=y)$$

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such that

(i) $P_{X,Y}(x,y) \geq 0$ (ii) $\sum_X \sum_Y P(x,y) = 1$

and $pr(a \leq X \leq b,\ c \leq Y \leq d)$

$$= \sum_{X=a}^{b} \sum_{Y=c}^{d} P(x,y)$$

- CdF for discrete Bivariate r.v (X,Y)

$$F(x,y) = pr(X \leq x,\ Y \leq y)$$

$$= \sum_{X \leq x} \sum_{Y \leq y} P(x,y)$$

- the marginal prob. mass fns for discrete Bivariate r.v (X,Y)

$P_X(x) = \sum_y P_{X,Y}(x,y)$ and $P_Y(y) = \sum_x P_{X,Y}(x,y)$

$\underline{Y}$

- For Continuous Bivariate distribution

* The Joint prob. density fn for continuous Bivariate r.v (X, Y) is $f_{X,Y}(x, y)$

s.t (i) $f_{X,Y}(x,y) \geq 0$

(ii) $$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,dx\,dy = 1$$

* CdF for Continuous Bivariate r.v (X, Y)

is $F(a,b) = pr(X \leq a, Y \leq b)$

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$$= \int_{-\infty}^{b}\int_{-\infty}^{a} f(x,y)\,dx\,dy$$

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$$\Rightarrow f(x,y) = \frac{\partial^2 F(x,y)}{\partial x \partial y}$$

Joint prob. density fn

- The marginal density fns for r.vs X and Y are

$$f_X(x) = \int_{-\infty}^{\infty} f(x,y)\,dy \quad \text{and} \quad f_Y(y) = \int_{-\infty}^{\infty} f(x,y)\,dx$$

respectively.

Some Definitions for Bivariate r.v (X, Y)

Defn (1): Covariance

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$$Cov(X,Y) = E\left[(X-\mu_X)(Y-\mu_Y)\right]$$

$$= E(XY) - \mu_X \mu_Y$$

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Defn (2): Correlation Coefficient

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$$\rho(X,Y) = \frac{Cov(X,Y)}{\sigma_X \, \sigma_Y}$$

σ_X: stand. dev. of X ; σ_Y: stand. dev. of Y

where $-1 \le \rho \le 1$.

Some props

(1) For independent r.vs X and Y, $Cov(X,Y) = 0$
and Consequently $\rho(X,Y) = 0$ i.e indep. r.vs $\Rightarrow \rho(X,Y)=0$

Note that For $\rho(X,Y) = 0$, It's not necessary for X and Y to be independent r.vs

(2) $Cov(X,X) = Var(X) = \sigma_X^2$

(3) $Cov(\alpha X, \alpha Y) = \alpha\beta \, Cov(X,Y)$ where $\alpha, \beta \in R$

(4) $Var(X+Y) = \sigma_X^2 + \sigma_Y^2 + Cov(X,Y)$