Introduction

Dr. Bander Almutairi

King Saud University

29 Jan 2014

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Examples: `http://www.mathsmutt.co.uk/files/diffeq.htm`.

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An *n-th linear* differential equation is of the form:

$$a_n(x)\frac{d^n y}{dx^n} + a_{n-1}(x)\frac{d^{n-1} y}{dx^{n-1}} + \ldots + a_2(x)\frac{d^2 y}{dx^2} + a_1(x)\frac{dy}{dx} + a_0(x)y = g(x),$$

or

$$f(x, y, \frac{dy}{dx}, \frac{d^2 y}{dx^2}, \ldots, \frac{d^n y}{dx^n}) = 0.$$

Origin of Differential Equations Solution:

1. Geometric Origin. Examples
   - $y = c_1 x + c_2$, striaght line.
   - $y = ce^{x^2/2}$, curve.
2. Physical Origin. Example: Free falling stone

$$\frac{d^2 s}{dt^2} = -g,$$

where s is the distance or height and g is acceleration due to gravity.

Q(1): Prove that $y = e^{2x}$ is a solution of the eqution $y'' + y' - 6y = 0$.
Q(2): Verify that $y = x^3 e^x$ is a solution of the differential equation $xy'' - 2(x+1)y' + (x+2)y = 0;\ x > 0$.
Q(3): verify that $F(x, y) = x^2 + y^2 - 4$ satisfyies an implicit solution of the differential equation $\frac{dy}{dx} = -\frac{x}{y}$, on the interval.
"We say a relation $F(x, y) = 0$ is an *implicit solution* of an ordinary differential equation on an interval I if the relation definds implicity a function $y = \phi(x)$ satisfy the differential equation.

Q(1): Eliminate the arbitrary constants c_1 and c_2 from the relation

$$y = c_1 e^{-2x} + c_2 e^{3x}.$$

Q(2): Eliminate the arbitrary constant $a$from the equation

$$(x - a)^2 + y^2 = a^2.$$

Q(3): Eliminate B and α from the relation

$$x = B\cos(\omega t + \alpha).$$

Q(4): Eliminate the arbitrary constant c from the family of curves

$$cxy + c^2 x + 4 = 0.$$