MATH107 Vectors and Matrices

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Definition

1. A *scalar* is a real number or a quantity that has magnitude only. Examples: length, temperature, area, volume.
2. A *vector* is a quantity that has magnitude and direction. Examples: Velocity, acceleration, force, momentum.

Note:

1- A vector is represented by directed line, for example, $\overrightarrow{PQ}$ represents a vector with initial point P and terminal point Q.

2- A has coordinates (a_1, a_2).

3- $\overrightarrow{0A}$ is the position vector, i.e. $a = \overrightarrow{0A}$.

4- $a = \langle a_1, a_2 \rangle$, a_1, a_2 are the components of vector a.

5- Magnitude of the vector a is

$$\|a\| = \sqrt{(a_1)^2 + (a_2)^2}.$$

6- If $A_1(a_1, b_1)$ and $A_2(a_2, b_2)$. We say (a_i, b_i) is a coordinate of A_i, where $i = 1, 2$. A vector a from A_1 to A_2 is

$$a = \overrightarrow{A_1A_2} = \langle a_2 - a_1, b_2 - b_1 \rangle .$$

Magnitude of the vector a is

$$\|a\| = \sqrt{(a_2 - a_1)^2 + (b_2 - b_1)^2}.$$

7- Let $a = \langle a_1, a_2 \rangle$ and $b = \langle b_1, b_2 \rangle$ be vectors in two dimension. Then

1. Addition and Subtraction: $a \pm b = \langle a_1 \pm b_1, a_2 \pm b_2 \rangle$.
2. scalar multiplication: $ka = \langle ka_1, ka_2 \rangle$.
3. Equality: $a = b$ if and only if $a_1 = b_1$ and $a_2 = b_2$.

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Example: Let $p(1,3), Q(2,5), W(1,1)$ be three points. Find: $a = \overrightarrow{PQ}$, $b = \overrightarrow{QW}$, $c = \overrightarrow{WP}$. Also, find $4a + 2b - 3c$, $\|a + 2b\|$. Find the unit vectors for $a, b, c, a + 2b$.

Definition

A *vector* a *in* 3*-space* is any ordered triple of real numbers

$$a = \langle a_1, a_2, a_3 \rangle \,,$$

where a_1, a_2 and a_3 are *the components* of the vector a.

Notes:

1- *The position vector* of a point $P(x, y, z)$ is

$$\overrightarrow{OP} = \langle x, y, z \rangle \,.$$

2- Let $a = \langle a_1, a_2, a_3 \rangle$ and $b = \langle b_1, b_2, b_3 \rangle$ be vectors in 3-space. Then

1. $a \pm b = \langle a_1 \pm b_1, a_2 \pm b_2, a_3 \pm b_3 \rangle$.
2. $ka = \langle ka_1, ka_2, ka_3 \rangle$.
3. $\mathbf{0} = \langle 0, 0, 0 \rangle$ is the zero vector.
4. $\|a\| = \sqrt{a_1^2 + a_2^2 + a_3^2}$.

3- If $\overrightarrow{OP_1}$ and $\overrightarrow{OP_2}$ are position vectors of points $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ then the vector $\overrightarrow{P_1P_2}$ is
$\overrightarrow{P_1P_2} = \overrightarrow{0P_2} - \overrightarrow{0P_1} = \langle x_2 - x_1, y_2 - y_1, z_2 - z_1 \rangle$.
Example: Given the points $P_1(1, -2, 3)$ and $P_2(-3, 2, -1)$. Find the vector a in V_3 that corresponds to $\overrightarrow{P_1P_2}$ and b that corresponds to $\overrightarrow{P_2P1}$.
Example: If $a = \langle -1, 3, 0 \rangle$ and $b = \langle -3, -2, -5 \rangle$. Find

1. $a + b$ and $b - a$.
2. $3a - 4b$ and $-2a - 3b$.
3. $\|a\|, \|b\|, \|3a - 4b\|$ and $\|4a\|$.
4. Find the unit vector that has same direction as a.
5. Find the vector that has the same direction as a and third the magnitude of a.
6. Find the vector that has the opposite direction of a and one-third the magnitude of a.

Note(1): If $b = k.a$, where k is scalar, then a and b are *parallel*.
Note(2): Three points lie on the same line, if two vectors from three points, they

(i) have same initial point;

(ii) are parallel.

Example: Use vectors to determine whether the points lie on a straight line, the points are $(1, -1, 5), (0, -1, 6)$ and $(3, -1, 3)$.

Example: If $a = \langle -6, -3, 6 \rangle$, find the vector that has

(i) the same direction of a and twice the magnitude of a.

(ii) the opposite direction of a and one-third the magnitude of a.

(i) the same direction of a and the magnitude 2.