

Exercise 1 :

- a) If $f = \ln(2x)$ and $g(x) = \int_1^{4x^2} (1+t^2)^{10} dt$. Since $f(\frac{1}{2}) = 0$ and $g(\frac{1}{2}) = 0$, then $F'(\frac{1}{2}) = 0$.

$$(F'(x) = \frac{1}{x} \int_1^{4x^2} (1+t^2)^{10} dt + 8x \ln(2x)(1+16x^4)^{10}.)$$

- b)

$$\begin{aligned} \int_0^2 3x^2 dx &= \lim_{n \rightarrow +\infty} 3 \frac{2}{n} \sum_{k=1}^n \frac{4k^2}{n^2} \\ &= \lim_{n \rightarrow +\infty} 24 \frac{1}{n^3} \left(\frac{n(n+1)(2n+1)}{6} \right) = 8. \end{aligned}$$

- c) $f(x) = \sin^4(x)$

k	x_k	$f(x_k)$	m	$mf(x_k)$
0	0	0	1	0
1	$\frac{\pi}{4}$	$\frac{1}{4}$	2	$\frac{1}{2}$
2	$\frac{\pi}{2}$	1	2	2
3	$\frac{3\pi}{4}$	$\frac{1}{4}$	2	$\frac{1}{2}$
4	π	0	1	0
				3

$$\int_0^\pi \sin^4(x) dx \approx \frac{3\pi}{8}.$$

Exercise 2

a) $f'(x) = \frac{1}{(\ln 2)(\sin^{-1}(x))\sqrt{1-x^2}}$ 2

b) $\int \frac{4^{-\ln(x)}}{x} dx \stackrel{t=-\ln(x)}{=} - \int 4^t dt = -\frac{4^{-\ln(x)}}{\ln 4} + c.$ 2 + 1

c) $y = e^{2x^2 \ln(x)}(x-1)^{\frac{3}{2}}, \ln(y) = 2x^2 \ln(x) + \frac{3}{2} \ln(x-1).$ 1
 $\frac{y'}{y} = 4x \ln(x) + 2x + \frac{3}{2(x-1)}$ and $y' = \left(4x \ln(x) + 2x + \frac{3}{2(x-1)}\right) y.$

1,5

0,5

Exercise 3 :

a)

$$\begin{aligned}\int \frac{2x+3}{\sqrt{4-x^2}} dx &= \int \frac{2x}{\sqrt{4-x^2}} dx + 3 \int \frac{dx}{\sqrt{4-x^2}} \\ &= -2\sqrt{4-x^2} + 3 \sin^{-1}\left(\frac{x}{2}\right) + c.\end{aligned}$$

b) $\int \frac{e^{\frac{x}{2}}}{7+e^x} dx \stackrel{t=e^{\frac{x}{2}}}{=} 2 \int \frac{dt}{7+t^2} = \frac{2}{\sqrt{7}} \tan^{-1}\left(\frac{x}{\sqrt{7}}\right) + c.$

c)

$$\begin{aligned}\int \frac{\sin(x)}{\sqrt{e^{\cos(x)}-1}} dx &\stackrel{t=\cos(x)}{=} - \int \frac{dt}{\sqrt{e^t-1}} \\ &\stackrel{u^2=e^t-1}{=} - \int \frac{2du}{1+u^2} = -2 \tan^{-1}(\sqrt{e^{\cos(x)}-1}).\end{aligned}$$

Or

$$\begin{aligned}\int \frac{\sin(x)}{\sqrt{e^{\cos(x)}-1}} dx &\stackrel{t=e^{\frac{1}{2}\cos(x)}}{=} -2 \int \frac{dt}{t\sqrt{t^2-1}} \\ &= -2 \sec^{-1}(e^{\frac{1}{2}\cos(x)}) + c.\end{aligned}$$