1 Mid Term Exam   **280-Math**   **1Semester (1440/1441)H**

Question 1$(2+2+1^*)$. (a) Determine $\sup E$ if $E = \{ x \in \mathfrak{R}, \ x^3 + 4x^2 < -3x \}$

(b) Determine $\inf E$ if $E = \{ x \in \mathfrak{R}, \ x^3 + 4x^2 > -3x \}$

$(c)^*$ Determine $\sup E$, $\min E$, $\inf E$ and $\max E$ if $E = \{ x \in [-4, 1], \ x^3 + 4x^2 + 3x < 0 \}$

Question 2$(1+2+4^*)$. (a) Show that the set $E = \{ \ \dfrac{m}{2m+3n} \ , \ n,m \in \mathbf{N} \}$ is bounded.

(b) Using the definition of inf find $\inf E$.

$(c)^*$ Using the definition of sup find $\sup E$.

Question 3 (3). (a) Find $\lim\limits_{n \to \infty} x_n$ if $x_n = \dfrac{2a^n - 3b^n}{a^n + 2b^n}$, $a > 0$; $b > 0$

Question 4 (3). Decide whether the set $E = \{ \dfrac{4^n}{n!} \ , \ n \in \mathbf{N} \}$ is bounded.

Question 5 (3). Using the $(\varepsilon - N)$ definition of convergence show that:

$$\lim\limits_{n \to \infty} \frac{n}{2n+100} = \frac{1}{2}$$

Question 6 (3). Find the sum of the following series if it converges:

$$\sum_{n=1}^{\infty} \frac{1}{n^2 + \dfrac{1}{3}n - \dfrac{2}{9}}$$

Question 7 (2+2+2). Determine whether each series converges or diverges:

(a) $\sum\limits_{n=1}^{\infty} (-1)^n \ln (1+\dfrac{1}{n})$ (b) $\sum\limits_{n=1}^{\infty} (e^{\frac{1}{n^2}} - 1)$ (c) $\sum\limits_{n=1}^{\infty} \dfrac{(-1)^n n}{e^{n^2+1}}$