

King Saud University  
Department of Mathematics

1 Mid Term Exam

280-Math

1Semester (1440/1441)H

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**Question 1** (2+2+1\*). (a) Determine  $\sup E$  if  $E = \{x \in \mathfrak{R}, x^3 + 4x^2 < -3x\}$

(b) Determine  $\inf E$  if  $E = \{x \in \mathfrak{R}, x^3 + 4x^2 > -3x\}$

(c)\* Determine  $\sup E$ ,  $\min E$ ,  $\inf E$  and  $\max E$  if  $E = \{x \in [-4, 1], x^3 + 4x^2 + 3x < 0\}$

**Question 2** (1+2+4\*). (a) Show that the set  $E = \left\{ \frac{m}{2m+3n}, n, m \in \mathbf{N} \right\}$  is bounded.

(b) Using the definition of  $\inf$  find  $\inf E$ .

(c)\* Using the definition of  $\sup$  find  $\sup E$ .

**Question 3** (3). (a) Find  $\lim_{n \rightarrow \infty} x_n$  if  $x_n = \frac{2a^n - 3b^n}{a^n + 2b^n}$ ,  $a > 0$ ;  $b > 0$

**Question 4** (3). Decide whether the set  $E = \left\{ \frac{4^n}{n!}, n \in \mathbf{N} \right\}$  is bounded.

**Question 5** (3). Using the  $(\varepsilon - N)$  definition of convergence show that:

$$\lim_{n \rightarrow \infty} \frac{n}{2n+100} = \frac{1}{2}$$

**Question 6** (3). Find the sum of the following series if it converges:

$$\sum_{n=1}^{\infty} \frac{1}{n^2 + \frac{1}{3}n - \frac{2}{9}}$$

**Question 7** (2+2+2). Determine whether each series converges or diverges:

(a)  $\sum_{n=1}^{\infty} (-1)^n \ln \left(1 + \frac{1}{n}\right)$       (b)  $\sum_{n=1}^{\infty} (e^{\frac{1}{n^2}} - 1)$       (c)  $\sum_{n=1}^{\infty} \frac{(-1)^n n}{e^{n^2+1}}$