King Saud University
Department of Mathematics

Home Assignment **280-Math** **2Semester (1440/1441)**

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Question1(3+3)**.** (a) Decide whether the set $E=\{2n+1+\frac{1}{2n-5}\ ,n\in \mathrm{N}\}$ is bounded.

(b) find $\sup E$ and $\inf E$.

Question2 (3+2)**.** (a) Using the $\varepsilon-$definition show that $\lim_{n\to\infty} x_n = 0$ if $x_n = \frac{1}{n}+\frac{\sin n}{n+1}$

(b) Decide whether the sequence $y_n = n+\frac{\cos n\sqrt{\pi}}{n}$ is Cauchy or not.

Question3 (2.5+2.5+2.5)**.** Find $\lim_{n\to\infty} x_n$ or show that it does not exist if:

a) $x_n = (-1)^n n^2(\cos\frac{2}{n}-1)$

b) $x_n = \frac{(n!)^2}{(2n)!}$

c) $x_n = \sum_{k=3}^{n}\frac{1}{k^2-3k+2}$

Question4 (2.5+2.5+2.5)**.** Determine whether the following series are convergent or divergent:

a) $\sum_{n=1}^{\infty}\frac{(n!)^2}{(2n)!}$

b) $\sum_{n=0}^{\infty}(-1)^n\frac{n}{e^{2n}}$

c) $\sum_{n=1}^{\infty}\left((\frac{2}{3})^n+(\frac{-1}{3})^n+\frac{3}{2}n\sin\frac{1}{n}\right)^n$

Question5 (6)**.** Find $\lim_{n\to\infty} x_n$ if $x_n = \frac{1}{n+1} + \frac{1}{n+2} + \frac{1}{n+3} + \cdots + \frac{1}{2n}$ or show that it is divergent.

Question6 (6)**.** prove that there is at least one real number c such that

$$c^3 = 3c + 1$$

Question7 (6)**.** prove that $x \le e^x \ \forall x \in \Re$

Question8 (6)**.** Decide whether the following improper integral is convergent or divergent: $I = \int_0^1 \frac{\sin x^2}{\sqrt{x^7}} dx$