p. 1

M - 107
TIME: 90min
TERM

KING SAUD UNIVERSITY **FULL MARKS: 50**

DEPARTMENT OF MATHEMATICS

(SEMESTER I, 1438-1439) FIRST MID-

Question: 1.(a) Solve the system of linear equations using reduced row echelon form

$$x+2y-3z+w=-2$$
$$3x-y-2z-4w=1 \qquad [8]$$
$$2x+3y-5z+w=-3$$

Soln

$$[A|b]=\left[\begin{array}{cccc|c}1&2&-3&1&-2\\3&-1&-2&-4&1\\2&3&-5&1&-3\end{array}\right]=\left[\begin{array}{cccc|c}1&0&-1&-1&0\\0&1&-1&1&-1\\0&0&0&0&0\end{array}\right]$$

$x-z-w=0$

$y-z+w=-1$

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$x=z+w$

$y=-1+z-w$

Let $z=t$, $w=s$, t and $s\in R$

$x=t+s$

$y=-1+t-s$

$z=t$

$w=s$

2

infinite many solutions

(b) For what values of λ does the following system of linear equations have (i) unique solution, (ii) no solution.

$$x+y+z=3$$
$$2x-y-z=1 \qquad [8]$$
$$3x+\lambda y+\lambda^2 z=7$$

Soln.

$$[A|b]=\left[\begin{array}{ccc|c}1&1&1&3\\2&-1&-1&1\\3&\lambda&\lambda^2&7\end{array}\right]=\left[\begin{array}{ccc|c}1&1&1&3\\0&-3&-3&-5\\0&\lambda-3&\lambda^2-3&-2\end{array}\right]$$

$$\left[\begin{array}{ccc|c}1&1&1&3\\0&1&1&5/3\\0&\lambda-3&\lambda^2-3&-2\end{array}\right]=\left[\begin{array}{ccc|c}1&1&1&3\\0&1&1&\frac{5}{3}\\0&0&\lambda^2-\lambda&-\frac{5}{3}(\lambda-3)-2\end{array}\right]$$

$-\frac{5}{3}\lambda+3$

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(a) Unique solution $\lambda\neq0$, $\lambda\neq1$

(b) No solution $\lambda=0$ or $\lambda=1$

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p. 2

Question: 2. (a) Let $M=\begin{bmatrix} 0 & 1 & 0 \\ -4 & 4 & 0 \\ -2 & 1 & 2 \end{bmatrix}$ be a matrix **[8]**

Find all values of λ such that matrix $M-\lambda I_3$ is invertible.

$$|M-\lambda I_3| = \begin{vmatrix} -\lambda & 1 & 0 \\ -4 & 4-\lambda & 0 \\ -2 & 1 & 2-\lambda \end{vmatrix}$$

$$= (2-\lambda)(\lambda-2)^2 = -(\lambda-2)^3$$ 6

$M-\lambda I_3$ is invertible iff $|M-\lambda I_3| \neq 0 \Rightarrow \lambda \neq 2$ 2

(b) Write the system of linear equations in matrix form AX = b. Find A^{-1} using Elementary matrix method, hence solve the system

$$x - y = 2$$
$$x - z = -1$$
$$-6x + 2y + 3z = 3$$

<u>Soln</u> **[8]**

$$\begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ -6 & 2 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} \qquad AX = b$$

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ -6 & 2 & 3 \end{bmatrix}$$ 2

$$[A \mid I] = \left[\begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ 1 & 0 & -1 & 0 & 1 & 0 \\ -6 & 2 & 3 & 0 & 0 & 1 \end{array}\right]$$

$$\equiv \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & -3 & -1 \\ 0 & 1 & 0 & -3 & -3 & -1 \\ 0 & 0 & 1 & -2 & -4 & -1 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} -2 & -3 & -1 \\ -3 & -3 & -1 \\ -2 & -4 & -1 \end{bmatrix}$$ 4

$$X = A^{-1}b = \begin{bmatrix} -2 & -3 & -1 \\ -3 & -3 & -1 \\ -2 & -4 & -1 \end{bmatrix}\begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} = \begin{bmatrix} -4 \\ -6 \\ -3 \end{bmatrix}$$ 2

p. 3

Question:3.(a) Verify the equation by using properties of determinant,

$$\begin{vmatrix} a+b & a & a \\ a & a+b & a \\ a & a & a+b \end{vmatrix} = b^2(3a+b) \qquad \textbf{[8]}$$

Soln

$$\begin{vmatrix} a+b & a & a \\ a & a+b & a \\ a & a & a+b \end{vmatrix} \overset{(R_3+R_2)+R_1}{=} \begin{vmatrix} 3a+b & 3a+b & 3a+b \\ a & a+b & a \\ a & a & a+b \end{vmatrix}$$

$$= (3a+b)\begin{vmatrix} 1 & 1 & 1 \\ a & a+b & a \\ a & a & a+b \end{vmatrix} \overset{C_2-C_1,\ C_3-C_1}{=} (3a+b)\begin{vmatrix} 1 & 0 & 0 \\ a & b & 0 \\ a & 0 & b \end{vmatrix}$$

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$= \boxed{b^2(3a+b)}$

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(b) Use cofactor method to find inverse of matrix A

$$A = \begin{bmatrix} -1 & 3 & 2 \\ 0 & -2 & 1 \\ 1 & 0 & 2 \end{bmatrix} \qquad \textbf{[10]}$$

Soln. matrix of cofactors

$$C = \begin{bmatrix} -4 & 1 & 2 \\ -6 & -4 & 3 \\ 7 & 1 & 2 \end{bmatrix}$$

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$\det A = 11$

$$\text{adj } A = C^T = \begin{bmatrix} -4 & -6 & 7 \\ 1 & -4 & 1 \\ 2 & 3 & 2 \end{bmatrix}$$

2

$$A^{-1} = \frac{1}{\det A}\text{adj } A = \frac{1}{11}\begin{bmatrix} -4 & -6 & 7 \\ 1 & -4 & 1 \\ 2 & 3 & 2 \end{bmatrix}$$

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$$L.H.S = \begin{vmatrix} a+b & a & a \\ a & a+b & a \\ a & a & a+b \end{vmatrix}$$

$$= \begin{vmatrix} 3a+b & 3a+b & 3a+b \\ a & a+b & a \\ a & a & a+b \end{vmatrix} \qquad (R_2+R_3)+R_1$$

$$= (3a+b)\begin{vmatrix} 1 & 1 & 1 \\ a & a+b & a \\ a & a & a+b \end{vmatrix}$$

$$= (3a+b)\begin{vmatrix} 1 & 1 & 1 \\ 0 & b & 0 \\ 0 & 0 & b \end{vmatrix} \qquad \begin{array}{l} -aR_1+R_2 \\ /\ -aR_1+R_3 \end{array}$$

$$= b^2(3a+b)\begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$= b^2(3a+b) = R.H.S \quad \#$$

ويوجد حلول اخرى ....