**King Saud university**
**Department of Mathematics**
**M - 203**
**(Differential and Integral Calculus)**
**Final Examination (Summer Semester 1434/1435)**
**Full Marks: 40 Time: 3 Hours**

**Q.** #1. **[Marks: 3+3+3+3=12]**
(a) Determine whether the sequence $\{\left(\frac{n+1}{n-1}\right)^n\}$ convergences or diverges and if it converges, find its limit.
(b) Determine the convergence or divergence of the series $\sum_{n=1}^{\infty} \frac{1}{2^n-1}$.
(c) Find the interval of convergence and the radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{(x+\pi)^n}{\sqrt{n}}$.
(d) Find the first three non-zero terms of a Taylor series for the function $f(x) = \cos x$ at $x = \pi/3$.
**Q.** #2. **[Marks: 3+3+3+3=12]**
(a) Evaluate the integral $\int_0^4 \int_{\sqrt{y}}^2 e^{x^3} dxdy$.
(b) Find the area of the portion of the surface given by the cone $z^2 = 4x^2 + 4y^2$ that is above the region in the first quadrant bounded by the line $y = x$ and the parabola $y = x^2$.
(c) Use cylindrical coordinates to evaluate the integral
$\int_0^a \int_0^{\sqrt{a^2-x^2}} \int_0^{a^2-x^2-y^2} x^2 dzdydx \ \ (a > 0)$.
(d) Find the mass of the solid enclosed between the two spheres $x^2+y^2+z^2 = 1$ and $x^2 + y^2 + z^2 = 4$ with density $\delta(x, y, z) = (x^2 + y^2 + z^2)^{-\frac{1}{2}}$.
**Q.** #3. **[Marks: 4+4+4+4=16]**
(a) Show that the following line integral is independent of path and find its value:
$\int_{(0,0)}^{(\pi,\pi)} (x + y)dx + (x - y)dy$.
(b) Use Green's theorem to evaluate the line integral
$\oint_C xydx + (x^2 + y^2)dy$,
where $C$ is the closed curve determine by $y = x$ and $y^2 = x$ with $0 \leq x \leq 1$.
(c) Use divergence theorem to evaluate the integral $\int\int_S \overrightarrow{F}.\overrightarrow{n}dS$, where $\overrightarrow{F} = 4x\overrightarrow{i} - 4y\overrightarrow{j} + z^2\overrightarrow{k}$ and $S$ is the surface of the region bounded by the cylinder $x^2 + y^2 = 4$ and the planes $z = 0$ and $z = 3$.
(d) Use Stokes' theorem to evaluate $\oint_C \overrightarrow{F}.d\overrightarrow{r}$, where $C$ is the boundary of the portion of $z = 4 - x^2 - y^2$ above the $xy$-plane oriented upward and $\overrightarrow{F}(x, y, z) = (x^2e^x - y)\overrightarrow{i} + \sqrt{y^2 + 1}\overrightarrow{j} + z^3\overrightarrow{k}$.