King Saud University, Department of Mathematics
Math 204 (3H), 40/40, Final Exam 7/3/36

Question 1[4,4] a) Determine and sketch the largest region of the \( xy \)-plane for which the following initial value problem has a unique solution

\[
\begin{aligned}
(x - 2)(x + 3)y' &= 4 \ln y \\
y(-5) &= 2.
\end{aligned}
\]

b) Test if the following equation is exact, if it is not, find the appropriate integrating factor and solve it.

\[(3x^2 + y)dx + (2x^2y - x)dy = 0.
\]

Question 2[4,4,5] a) Solve the differential equation

\[
\frac{dy}{dx} = \sqrt{3 + x + y},
\]

b) Solve the initial value problem

\[
\begin{aligned}
xy' - 2(1 + x + \sqrt{y})y &= 0, \quad x > 0, y > 0 \\
y(1) &= 1.
\end{aligned}
\]

c) A building loses heat in accordance with Newton's law of cooling. Assume the inside temperature is 70\(^0\)F when the heating system fails. After 2 hours the inside temperature drops to 40\(^0\)F. If the external temperature is 20\(^0\)F, compute the interior temperature after 4 hours.

Question 3[4,5] a) Use the variation of parameters method to solve the differential equation

\[y'' - 2y' + y = \frac{e^x}{x^2 + 1}.
\]

b) Use power series method to solve the nonhomogeneous equation

\[y'' + xy' - 2y = x,
\]

about the ordinary point \( x = 0 \).

Question 4[5,5] a) Let

\[f(x) = \begin{cases} 
1 + x, & -1 \leq x \leq 0 \\
-1 + x, & 0 < x \leq 1 
\end{cases}
\]

where \( f(x + 2) = f(x) \ \forall x \in \mathbb{R}. \) Sketch the graph of \( f(x) \) on \((-1,1)\) and find its Fourier series.

b) Find the Fourier integral of the function

\[f(x) = \begin{cases} 
-2, & -1 \leq x \leq 0 \\
1, & 0 < x \leq 1 \\
0, & |x| > 1
\end{cases}
\]

and deduce that

\[\int_{0}^{\infty} \frac{(3 - 4 \cos \lambda) \sin \lambda}{\lambda} d\lambda = \frac{\pi}{2}.\]