Question 1[4,4]. a) Find and sketch the largest region of the $xy$-plane for which the initial value problem
\[
\begin{aligned}
(x^2 + y^2) \frac{dy}{dx} &= x\sqrt{y - 1} \\
y(-2) &= 4.
\end{aligned}
\]
has a unique solution.

b) Find the solution of the differential equation:
\[
\frac{dy}{dx} (y - 1)\sqrt{x^2 + 1} + x^3 + x = y(x^3 + x) ; \quad y \neq 1.
\]

Question 2[4,4]. a) Solve the following differential equation
\[
(3xy - x + y^2) + (x^2 + xy) \frac{dy}{dx} = 0 ; \quad x > 0 , \quad x + y \neq 0.
\]

b) Find the solution of the initial value problem
\[
\begin{aligned}
[x \cos^2(\frac{x}{y}) - y] \ dx + x \ dy &= 0 \\
y(1) &= \frac{\pi}{4}.
\end{aligned}
\]

Question 3[4]. Find the general solution of the differential equation
\[
y^3 \frac{dy}{dx} + xy^3 = xe^{-x^2} ; \quad x > 0 , \quad y \neq 0.
\]

Question 5[5]. Find the family of orthogonal trajectories for the family of curves
\[
y = e^{C \sin x} ; \quad 0 < x < \frac{\pi}{2},
\]
where $C$ is an arbitrary constant such that $C \neq 0$. 