Question 1[4,4]. a) Determine the region in the xy-plane for which the following differential equation

\[(1 - y^2) \frac{dy}{dx} = xe^x,\]

would have a unique solution through the origin \((0, 0)\).

b) Find the solution of the differential equation:

\[\frac{dy}{dx} - 2xy = e^x(1 - 2x).\]

Question 2[4,4]. a) Verify that the differential equation

\[
\cos x dx + \left(1 + \frac{2}{y}\right) \sin x dy = 0, \quad y \neq 0,
\]

is not exact. Find a suitable integrating factor to convert it to an exact equation, and then solve it.

b) Solve the initial value problem

\[
\begin{cases}
\frac{dy}{dx} = \frac{x}{y} + \frac{y}{x} \\
y(1) = 2
\end{cases} \quad x \neq 0, \quad y \neq 0
\]

Question 3[4]. Find the general solution of the differential equation

\[
\frac{dy}{dx} + \tan x \cdot y = \frac{(4x + 5)^2}{2 \cos x} y^3, \quad -\frac{\pi}{2} < x < \frac{\pi}{2}.
\]

Question 5[5]. A thermometer is removed from a room where the air temperature is 70\(^\circ\)F to outside where the temperature is 10\(^\circ\)F. After 1/2 minute the thermometer reads 50\(^\circ\)F. What is reading at \(t = 1\) minute?. How long will it take for the thermometer to reach 15\(^\circ\)F.