King Saud University: Mathematics Department Math-254
Summer Semester 1430-31 H Midterm Examination
Maximum Marks = 35 TIMe: 120 Mins.

Name of the Student:—————————— I.D. No. ——————

Name of the Teacher:—————————— Section No. ——————

The Answer Table for Q.1 to Q.10 : Marks: 2 for each one ($2 \times 10 = 20$)

Ps. : Mark {a, b, c or d} for the correct answer in the box.

Q. No.	1	2	3	4	5	6	7	8	9	10
a,b,c,d										

Question No.	Marks
Q. 1 to Q. 10	
Q. 11	
Q. 12	
Q. 13	
Total Marks	

Question 1: The second approximation to the root of the equation $x^3 = x^2 + 1$ in $[1, 2]$ by the Bisection method is:

(a) $c_2 = 1.75$ (b) $c_2 = 1.5$ (c) $c_2 = 1.25$ (d) $c_2 = 1.625$

Question 2: The equivalent form $f(x) = 0$ of the nonlinear equation $g(x) = \dfrac{2x^3 - 2}{3x^2 - 2}$ is:

(a) $x^3 + 3x + 2 = 0$ (b) $x^3 - 2x + 2 = 0$ (c) $x^3 - 3x + 2 = 0$ (d) $x^3 - 2x - 2 = 0$

Question 3: Newton's iterative formula for approximation to the square root of a real number R is:

(a) $x_{n+1} = \dfrac{x_n}{2R}$ (b) $x_{n+1} = \dfrac{1}{2}(3x_n - \dfrac{R}{x_n})$ (c) $x_{n+1} = \dfrac{3Rx_n}{2}$ (d) $x_{n+1} = \dfrac{1}{2}(x_n + \dfrac{R}{x_n})$

Question 4: The first approximation of the root of $x^2 = 4$ using Newton's iterative formula, if $x_0 = 3$; is:

(A) 2.067 (b) 1.5 (c) 3.5 (d) 2.167

Question 5: The first approximation of the root of $x^3 + 4x^2 = 10$ using modified Newton's iterative formula, if $x_0 = 1.5$; is:

(a) $x_1 = 1.46$ (b) $x_1 = 1.36$ (c) $x_1 = 1.56$ (d) $x_1 = 1.66$

Question 6: If the iterative scheme $x_{n+1} = ax_n^2 + \dfrac{2b}{x_n} - 8,\ n \geq 0$ converges quadratically to 1, then the values of a and b are:

(a) -2 and 2 (b) 1 and 1 (c) 3 and 3 (d) -3 and -2

Question 7: The order of convergence of Newton's method to the root $\alpha = 3$ of the equation $(x-3)^3 e^{(x-3)} = 0$ is:

(a) of order 2 (b) of order 4 (c) of order 3 (d) of order 1

Question 8: If the linear system $\begin{aligned} 6x - 4y &= 2 \\ -3x + 2y &= k \end{aligned}$ has infinitely many solutions, then the value of k is:

(a) $k = 4$ (b) $k = -1$ (c) $k = -4$ (d) $k = 1$

Question 9: Determinant of the matrix $A = \begin{pmatrix} 1 & 2 & 4 \\ 1 & 3 & 3 \\ 2 & 2 & 2 \end{pmatrix}$ by LU decomposition ($l_{ii} = 1$) is:

(a) 18 (b) 28 (c) 8 (d) -8

Question 10: The value of α for which the matrix $A = \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \\ -1 & 1 & \alpha \end{pmatrix}$ is singular, is:

(a) $\alpha = 1$ (b) $\alpha = 2$ (c) $\alpha = 3$ (d) $\alpha = 1.5$

Question 11: Develop an iterative formula for the root of any positive number N using Scant method. Then use it to find the first approximation of the fifth root of 32 using $x_0 = 1.0$ and $x_1 = 1.5$. Find the absolute error. [5 points]

Question 12: Use Newton's method to find the first approximation to the roots of the following nonlinear system with $x_0 = 1$ and $y_0 = -1$: [5 points]

$$\begin{aligned} x^3 + 3y^2 &= 2 \\ x^2 + 2y &= -2 \end{aligned}$$

Question 13: Use LU decomposition method with **Crout's method** ($u_{ii} = 1$) to find the unique solution to the following linear system: [5 points]

$$\begin{aligned} x + 2y + 3z &= 1 \\ 6x + 5y + 4z &= -1 \\ 2x + 5y + 6z &= 5 \end{aligned}$$

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Q1) $f(x) = x^3 - x^2 - 1 = 0 \quad [1,2]$

$f(1) = (-) \quad , f(2) = (+)$

$x_1 = \frac{a+b}{2} = \frac{1+2}{2} = 1.5 \quad , f(1.5) = \frac{1}{8} \; (+)$

$x_2 = \frac{x_1 + a}{2} = \frac{1.5+1}{2} = 1.25$ $\boxed{C_2 = 1.25}$

Q2) $g(x) = \frac{2x^3 - 2}{3x^2 - 2}$

$x = \frac{2x^3 - 2}{3x^2 - 2}$

$(3x^2 - 2)x = 2x^3 - 2 \Rightarrow 3x^3 - 2x = 2x^3 - 2$

$\boxed{f(x) = x^3 - 2x + 2 = 0}$ (b)

Q3) $x = \sqrt{R}$ newton's $\Rightarrow x_{n+1} = x_0 - \frac{f(x_n)}{f'(x_n)}$

$f(x) = x^2 - R = 0$

$f'(x) = 2x = 0$

$x_{n+1} = x_n - \frac{(x_n^2 - R)}{(2x_n)}$

$= x_n - \left[\frac{x_n^2}{2x_n} - \frac{R}{2x_n}\right] \Rightarrow x_n - \frac{x_n}{2} + \frac{R}{2x_n}$

$\boxed{x_{n+1} = \frac{1}{2}\left[x_n + \frac{R}{x_n}\right]}$ (d)

Q4) $f(x) = x^2 - 4 = 0 \quad x_0 = 3$

$f'(x) = 2x = 0 \quad x_1 = 3 - \frac{(3^2 - 4)}{(2 \times 3)} = \boxed{2.167}$ (d)

Q5) ~~$f(x) = x^3 + 4x^2 - 10 = 0$~~

~~$f'(x) = 3x^2 + 8x = 0$~~

~~$f''(x) = 6x + 8 = 0$~~

$x_0 = 1.5$ ~~$x_{n+1} = x_n - \frac{f(x_n) \cdot f'(x_n)}{f'(x_n)^2 - f(x_n) \cdot f''(x_n)}$~~ $x = \sqrt{x^3 + 4x^2 - 10}$

~~$x_1 = 1.5 - \frac{(1.5^3 + 4(1.5)^2 - 10)(3(1.5)^2 + 8(1.5))}{(3(1.5)^2 + 8(1.5))^2 - (1.5^3 + 4(1.5)^2 - 10)(6(1.5) + 8)} =$~~

$f(x) = x^3 + 3x^2 - 10$

$f'(x) = 3x^2 + 6x$

$f''(x) = 6x + 6$

$x_1 = 1.5 \frac{0.125 \times 15.75}{(15.75)^2 - (0.125 \times 15)} = \boxed{1.49 \approx x_1 = 1.46 \text{ (a)}}$

Q6)

$f(x) = (x-3)^3 e^{(x-3)} = 0$

$f(3) =$???

Q7) order of 4 (b)

Q8) $A = \begin{bmatrix} 6 & -4 & 2 \\ -3 & 2 & k \end{bmatrix} = \begin{bmatrix} 6 & -4 & 2 \\ 0 & 0 & (k+1) \end{bmatrix}$

$\frac{1}{2}R_1 + R_2$

$0x_2 = (k+1)$

so, $\boxed{k = -1}$ (b)

Q9) $A = \begin{bmatrix} 1 & 2 & 4 \\ 1 & 3 & 3 \\ 2 & 2 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 4 \\ 0 & 1 & -1 \\ 0 & -2 & -6 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 4 \\ 0 & 1 & -1 \\ 0 & 0 & -8 \end{bmatrix} = U$

$-R_1 + R_2,\ -2R_1 + R_3$ $\quad$ $2R_2 + R_3$

$L = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 2 & -2 & 1 \end{bmatrix} \Rightarrow LU = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 2 & -2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 4 \\ 0 & 1 & -1 \\ 0 & 0 & -8 \end{bmatrix}$

$|A| = |LU| = |L||U| \Rightarrow L = (1)(1)(1) = 1, |U| = (1)(1)(8) = 8$

$LU = (1)(8) = \boxed{8}$ (c)

Q10) $A = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \\ -1 & 1 & \alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \\ 0 & 1 & (\alpha-1) \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & (\alpha-2) \end{bmatrix}$

$(1)R_1 + R_3$ $\quad$ $-R_2 + R_3$

$0x_3 = (\alpha - 2) \Rightarrow \boxed{\alpha = 2}$ (b)

Q12)

$$\begin{bmatrix} x_{n+1} \\ y_{n+1} \end{bmatrix} = \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} - \begin{bmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial y} \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} \quad , x_0 = 1, \; y_0 = -1$$

$$\left.\begin{aligned} f(x_1) &= x^3 + 3y^2 - 2 = 0 \\ f(x_2) &= x^2 + 2y + 2 = 0 \end{aligned}\right\} \quad \frac{\partial f_1}{\partial x} = 3x^2 \; , \; \frac{\partial f_1}{\partial y} = 6y \qquad \frac{\partial f_2}{\partial x} = 2x \; , \; \frac{\partial f_2}{\partial y} = 2$$

$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix} - \begin{bmatrix} 3(1)^2 & 6(-1) \\ 2(1) & 2 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix} - \begin{bmatrix} 0 \\ 6 \end{bmatrix} = \begin{bmatrix} 1 \\ -7 \end{bmatrix} \Rightarrow \begin{aligned} x_1 &= 1 \\ y_1 &= -7 \end{aligned}$$

Q13)

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 6 & 5 & 4 \\ 2 & 5 & 6 \end{bmatrix} , \; b = \begin{bmatrix} 1 \\ -1 \\ 5 \end{bmatrix} \quad , \quad Ax = b \; , \; Ly = b \; , \; y = Ux$$

$-6R_1 + R_2 , \; -2R_1 + R_3$

$$= \begin{bmatrix} 1 & 2 & 3 \\ 0 & -7 & -14 \\ 0 & 1 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 1 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & -4 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} = U$$

$-\frac{1}{7}R_2 \qquad -R_2 + R_3 \qquad -\frac{1}{4}R_3$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 6 & -7 & 0 \\ 2 & 1 & -4 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 5 \end{bmatrix}$$

$\boxed{y_1 = 1}$, $6(1) - 7y_2 = -1$, $\boxed{y_2 = 1}$, $2 + 1 - 4y_3 = 5$, $\boxed{y_3 = -\frac{1}{2}}$

$$\begin{bmatrix} 1 \\ 1 \\ -\frac{1}{2} \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$\boxed{x_3 = -\frac{1}{2}}$, $\boxed{x_2 = 2}$, $\boxed{x_3 = -1.5}$