King Saud University: Mathematics Department Math-254
First Semester 1431-32 H Final Examination
Maximum Marks = 50 Time: 180 mins.

Name of the Student:——————————— **I.D. No.** ——————————-

Name of the Teacher:——————————— **Section No.** ——————————

The Answer Table for Q.1 to Q.15 : Marks: 2 for each one ($2 \times 15 = 30$)

Ps. : Mark {a, b, c or d} for the correct answer in the box.

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
a,b,c,d															

Quest. No.	Marks
Q. 1 to Q. 15	
Q. 16	
Q. 17	
Q. 18	
Q. 19	
Total	

Question 1: The error bound for the 5^{th} approximation to the solution of the nonlinear equation $f(x) = 0$ in $[1.5, 2]$ using bisection method is:

(a) $\frac{1}{64}$ **(b)** $\frac{1}{32}$ **(c)** $\frac{1}{8}$ **(d)** $\frac{1}{16}$

Question 2: if the root of the nonlinear equation $f(x) = 0$ in $[0.5, 2]$ is a fixed point of the equation $g(x) = \sqrt{2-x}$, then $f(x) = 0$ is:

(a) $x^2 + x - 2 = 0$ **(b)** $\frac{x}{\sqrt{2-x}} - x = 0$ **(c)** $\frac{\sqrt{2-x}}{x} - x = 0$ **(d)** $x^2 - x + 2 = 0$

Question 3: The rate of convergence of Newton's method to the root $\alpha = 0$ of the equation $\cos x - 1 - 0.5x^2 = 0$ is:

(a) order 1 **(b)** order 2 **(c)** order 3 **(d)** order 4

Note: The following information will be used in Questions 4 to 6:

$$A = \begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}, \quad A^{-1} = \begin{bmatrix} 0.3 & -0.2 \\ -0.1 & 0.4 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}.$$

Question 4: The solution of the linear system $A\mathbf{x} = \mathbf{b}$ using LU-decomposition ($l_{ii} = 1$) is:

(a) $[1.1, -0.7]^T$ **(b)** $[0.1, -0.7]^T$ **(c)** $[1.1, 0.7]^T$ **(d)** $[-1.1, -0.7]^T$

Question 5: The relative error with respect to the approximate solution $\hat{\mathbf{x}} = [0.4, -0.6]$ for l_∞-norm is bounded by:

(a) 2.6 **(b)** 2.7 **(c)** 2.8 **(d)** 2.9

Question 6: Using Jacobi iteration method with the initial approximation $[0, 0]^T$, the error bound $\|\mathbf{x} - \mathbf{x}^{(4)}\|$ is:

(a) $\frac{3}{32}$ **(b)** $\frac{3}{22}$ **(c)** $\frac{3}{26}$ **(d)** $\frac{3}{16}$

Question 7: If the best approximation of $f(1.5)$ using the Newton's quadratic interpolating polynomial is 7 and $f[1, 2, 3, 4] = 8$, then the Newton's cubic polynomial $p_3(1.5)$ gives:

(a) 10.0 **(b)** 9.0 **(c)** 12.0 **(d)** 11.0

Question 8: Let $f(x) = \ln(x + 2)$ be given at the points $-1, 0, 4$, then the upper bound in approximating $\ln 3$ using a quadratic interpolating polynomial is:

(a) 2.0 **(b)** 1.0 **(c)** 3.0 **(d)** 4.0

Question 9: If a function $f(x)$ satisfies the conditions $f[-1,1] = 1, f'(1) = 5, f'(-1) = -1$, then $f[1,-1,1]$ equals:

(a) 2.0 **(b)** 3.0 **(c)** 4.0 **(d)** 5.0

Question 10: If $S(x) = \begin{cases} cx - 2, & \text{if } 0 \le x \le 1 \\ (4-c)x, & \text{if } 1 \le x \le 2 \end{cases}$ is a linear spline of a function $f(x)$, then the value of c is:

(a) 3.0 **(b)** 2.0 **(c)** 4.0 **(d)** 1.0

Note: The following information will be used in Questions 11 to 13:

x	0.0	0.1	0.2	0.3	0.4	0.45	0.5
$f(x)$	−2.0	0.0	3.0	5.0	8.0	10.0	14.0

Question 11: The best approximate value of $f'(0.3)$ using 3-point difference formula is:

(a) 25.0 **(b)** 20.0 **(c)** 30.0 **(d)** 35.0

Question 12: The best approximate value of $f''(0.4)$ is:

(a) 300 **(b)** 250 **(c)** 350 **(d)** 400

Question 13: The best approximate value of $\int_0^{0.5} f(x)\,dx$ is:

(a) 2.2 **(b)** 1.8 **(c)** 2.0 **(d)** 1.6

Question 14: The error bound in approximating $\int_0^1 \frac{15}{x+1}\,dx$ using the composite Trapezoidal rule with $n = 5$ is:

(a) 0.1 **(b)** 0.1×10^{-1} **(c)** 0.1×10^{-2} **(d)** 0.1×10^{-3}

Question 15: For IVP $y' + 3y = 4,\ y(0) = 5$, the approximate value of $y(0.1)$ using Taylor's method of order two when $n = 1$ is:

(a) 4.065 **(b)** 4.650 **(c)** 4.560 **(d)** 4.506

Question 16: Show that $\alpha = 1$ is the root of the nonlinear equation [5 points]

$$x^4 - x^3 - 3x^2 + 5x = 2.$$

Use quadratic convergent method to find its first approximation $x^{(1)}$ if $x^{(0)} = 0.5$.

Question 17: Solve the following system of linear equations using the Gaussian elimination with **partial pivoting** [5 points]

$$\begin{aligned} x_1 &+ x_2 &+ x_3 &= 1 \\ 2x_1 &+ 3x_2 &+ 4x_3 &= 3 \\ 4x_1 &+ 9x_2 &+ 16x_3 &= 11 \end{aligned}$$

Question 18: Let $x_0 \in (a, b)$, where$f \in C^2[a, b]$ and that $x_1 = x_0 + h \in (a, b)$ for some $h \neq 0$, then show that [5 points]

$$f'(x_0) \approx \frac{f(x_0 + h) - f(x_0)}{h}.$$

Use the above derived formula to find the approximate value of the derivative $f'(2.5)$ of the function $f(x) = (x + 1)\ln(x + 1)$, with $h = 0.05$.

Question 19: How many subintervals approximate the integral $\int_0^2 \frac{1}{x+4}\, dx$, to an accuracy 10^{-5} using the Simpson's rule ? Also, compute the approximation. [5 points]