Math 316

First Midterm Exam 1440, 1st semester

Name: ID:

Q1 Prove or disprove each of the following statements:

(a) The set $\{|x|, x\}$ is linearly independent on $[-1, 0]$.

(b) If X is an inner product space and $\{x, y\}$ is orthogonal in X, then $\|x + y\|^2 = \|x\|^2 + \|y\|^2$.

Q2 Consider the sequence of functions

$$f_n(x) = e^{-nx}, \qquad x \in (0, \infty)$$

where $n \in \mathbb{N}$.

(a) Show that $f_n \in \mathcal{L}^2(0, \infty)$ for all $n \in \mathbb{N}$.

(b) Find the limit $f(x)$ of $f_n(x)$ as $n \to \infty$.

(c) Does $f_n(x)$ converge to $f(x)$ uniformly? Justify your answer.

(d) Does $f_n(x)$ converge to $f(x)$ in $\mathcal{L}^2(0, \infty)$? Justify your answer.

Q3. Consider the set $S = \{1, \sin \pi x\}$ in $\mathcal{L}^2(0, 2)$

(a) Show that S is orthogonal in $\mathcal{L}^2(0, 2)$.

(b) Determine the coefficients α_i in the linear combination $\alpha_1 + \alpha_2 \sin \pi x$ which gives the best approximation in $\mathcal{L}^2(0, 2)$ of the function $f(x) = x$, $x \in (0, 2)$.

Q4 Consider the eigenvalue problem

$$u'' + 2u' + \lambda u = 0, \qquad x \in [0, 1], \tag{1}$$
$$u(0) = 0, \qquad u(1) = 0$$

(a) Find the eigenvalues and eigenfunctions of problem (1).

(b) Show that L is not a self-adjoint operator.

(c) Transform L into a self-adjoint operator.

(d) Write the orthogonality relation between the eigenfunctions of problem (1).

Good Luck

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