M555 First Semester 1427/1428 H Time: 90 minutes

First Midterm Exam

Question 1: (4 points)

Solve the following problem graphically

$$\begin{aligned} \max z &= 4x_1 + 4x_2 \\ \text{s.t.} \quad x_1 + x_2 &\leq 8 \\ -2x_1 + 3x_2 &\leq 6 \\ 2x_1 - x_2 &\leq 2 \\ x_1, x_2 &\geq 0. \end{aligned}$$

Question 2:(9 points)

Use the two phase method to find the optimal solution to the following LP:

$$\begin{aligned} \max z &= 4x_1 + 2x_2 \\ \text{s.t.} \quad 2x_1 + x_2 &\leq 6 \\ x_1 + x_2 &\geq 4 \\ x_1, x_2 &\geq 0. \end{aligned}$$

Question 3: (7 points)

If the following LP:

$$\begin{aligned} \max z &= 2x_1 + x_2 \\ \text{s.t.} \quad 3x_1 + x_2 &\leq 15 \\ 3x_1 + 2x_2 &\leq 18 \\ x_1, x_2 &\geq 0 \end{aligned}$$

has the optimal solution $z^* = 11$ at $x_1^* = 4, x_2^* = 3$, and $BV = \{x_1, x_2\}$, find the values of Δ such that the optimal solution remains optimal, and find the value of the new solution x_B^* and z^* in each of the following cases.

1. If $b_1 = 15$ is changed to $b_1 = 15 + \Delta$.
2. If $c_2 = 1$ is changed to $c_2 = 1 + \Delta$.

Question 4: (5 points)

If we have a max problem that has the following region

If $z(x) \geq 0 \forall x \in \Omega$, and the slope of the line $z = c$ equals $-1/5$, the slope of L_1 equals $-2/5$ and the slope of L_2 equals $-3/4$.

1. Which point is the optimal bfs.
2. If $z^* = 40$, find the LP problem.