Second Midterm Exam

Question 1: (9 points)

Solve the following problems by finding the optimal solution and the range of λ for which the optimal solution remains optimal.
(a)

$$\begin{aligned}\max z =&(2-3\lambda)x_1+(3+4\lambda)x_2\\ \text{s.t.}\quad & x_1+x_2\le 6\\ & x_1+2x_2\le 8\\ & x_1,x_2\ge 0.\end{aligned}$$

Use the optimal tableau to find the solution if $\lambda = 0.3$.
(b)

$$\begin{aligned}\max z =&2x_1+3x_2\\ \text{s.t.}\quad & x_1+x_2\le 6-3\lambda\\ & x_1+2x_2\le 8+4\lambda\\ & x_1,x_2\ge 0.\end{aligned}$$

Use the optimal tableau to find the solution if $\lambda = 0.3$.

Question 2: (7 points)

(a) Solve the following integer problem.

$$\begin{aligned}\max z =&4x_1+3x_2\\ \text{s.t.}\quad & 2x_1+x_2\le 6\\ & x_1+x_2\le 4\\ & x_1,x_2 \text{ are nonnegative integers.}\end{aligned}$$

(b) Use branch and bound to solve the following integer problem.

$$\begin{aligned}\max z =&2x_1+3x_2\\ \text{s.t.}\quad & x_1+3x_2\le 19\\ & x_1+x_2\le 10\\ & x_1,x_2 \text{ are nonnegative integers.}\end{aligned}$$

Question 3: (9 points)

(a) If we have the following LP

$$\begin{aligned}\max z = -3x_1+x_2+2x_3&\\ \text{s.t.}\qquad\qquad x_2+2x_3&\le 3\\ -x_1\qquad\quad +3x_3&\le -1\\ -2x_1-3x_2\qquad\quad&\le -2\\ x_1,x_2,x_3&\ge 0\end{aligned}$$

(1) Find the dual problem and show that it has the same feasible region Ω.

(2) Show that $\Omega \neq \phi$, then find the solution of the dual problem without using the simplex method.

(b) Consider the following LP:

$$\begin{aligned}
\max z &= c_1x_1 + c_2x_2 \\
\text{s.t.} \quad 3x_1 + 4x_2 &\leq 6 \\
2x_1 + 3x_2 &\leq 4 \\
x_1, x_2 &\geq 0 \ .
\end{aligned}$$

you are given that the optimal tableau for this LP is

$$\begin{aligned}
z \qquad\quad + \ s_1 + 2s_2 &= r \\
x_1 \quad + 3s_1 - 4s_2 &= 2 \\
x_2 - 2s_1 + 3s_2 &= 0
\end{aligned}$$

Without doing any pivots, determine c_1, c_2 and r.

Good Luck

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