FIRST SEMESTER FINAL EXAMINATION, 1442
Dept. Math., College of Science, KSU
Math: 107 Full Mark: 40 Time: 3 Hours

[**Marks: Q1:[4+3] Q2:[3+2+3] Q3:[3+3+3] Q4:[2+2+4] Q5:[3+3+2]**]

Q1. (a) Consider the following linear system where λ is a real number:

$$\begin{aligned} 2x + 3y + (\lambda+2)z &= 5 \\ x + y + z &= 2 \\ 4\lambda x + 3\lambda y + 3z &= 8\lambda - 3 \end{aligned}$$

Determine for which values of λ this system has (i) no solution, (ii) a unique solution, and (iii) infinitely many solution.

(b) Let

$$A = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 1 & 4 \\ -1 & 6 & 2 \end{bmatrix}$$

Apply adjoint method to find the inverse A^{-1} of the matrix A.

Q2. (a) Let $\vec{a} = \langle -2, 1, 2 \rangle$, $\vec{b} = \langle 1, -2, 2 \rangle$, and $\vec{c} = \langle 2, 2, 1 \rangle$. Show that $\text{Com}_{\vec{b}}\, \vec{a} = \text{Com}_{\vec{c}}\, \vec{b} = \text{Com}_{\vec{a}}\, \vec{c}$.

(b) Find the parametric equations of the line of intersection of the following planes:

$$\begin{aligned} P_1 :&\quad 2x + y + 4z = 8 \\ P_2 :&\quad x + 3y - z = -1 \end{aligned}$$

(c) Identify the quadratic surface $2x^2 + 3y^2 - 6z = 0$. Then sketch the surface by finding its traces on the coordinate planes.

Q3. (a) If the parametric equation of a circle of radius a is given by $C : x = a\cos t,\ y = a\sin t$, then show that the curvature $\kappa = \frac{1}{a}$.

(b) Find the unit tangent vector to the curve $x = t^2 + 1,\ \ y = 4t - 3,\ \ z = 2t^2 - 6t$ at the point when $t = 2$.

(c) The position vector of a moving particle at time t along a curve C is $\vec{r}(t) = t\vec{i} + t^2\vec{j} + t^3\vec{k}$, $1 \leq t \leq 4$. Find the tangential and normal components of acceleration at time t.

Q4. (a) Show that $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^6 y + x^2 y^3}{(x^4 + y^2)^2}$ does not exist .

(b) Find $\dfrac{dw}{dt}$ if $w = \ln(u + v)$, $u = e^{-3t}$, $v = t^5 - t^2$.

(c) Find the directional derivative of the function $f(x, y) = \dfrac{x - y}{xy + 2}$ at $(1, 1)$ in the direction of the vector $\vec{a} = 12\vec{i} + 5\vec{j}$; what is the maximum rate of change of this function at $(1, 1)$?

Q5. (a) Find the equation of tangent plane to the surface $z = x^3 - 12xy + 8y^3$ at $P(2, -1, 24)$.

(b) Find the extrema and saddle points of $f(x, y) = x^3 - 12xy + 8y^3$, if any.

(c) Use Lagrange multipliers to find the extrema of $f(x, y, z) = x^2 + y^2 + z^2$ subject to the constraint $x + y + z = 36$.