① السؤال الأول ①

$$\int_1^3 (1-2x)\,dx$$

$$\Delta x = \frac{3-1}{n} = \frac{2}{n} \quad ①$$

$$f(x) = 1-2x, \quad x_i = 1 + \frac{2}{n} i$$

$$f(x_i) = -1 - \frac{4}{n} i \quad ①$$

$$\sum_{i=1}^{n} f(x_i)\,\Delta x_i = \sum_{i=1}^{n} -\left(1 + \frac{4}{n} i\right)\frac{2}{n} = -\frac{2}{n}\left[\sum_{i=1}^{n} 1 + \frac{4}{n}\sum_{i=1}^{n} i\right]$$

$$= -\frac{2}{n}\left[n + \frac{4}{n}\,\frac{n(n+1)}{2}\right] = -2 - 4\left(1+\frac{1}{n}\right) \quad ①$$

$$\therefore \int_1^3 (1-2x)\,dx = \lim_{n\to\infty} -6 - \frac{4}{n} = \underline{\underline{-6}}$$

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② $\int_a^b f(x)\,dx = (b-a) f(c)$ ①

$$\int_0^3 (4x - x^2)\,dx = 3(4c - c^2)$$

$$\left[2x^2 - \frac{x^3}{3}\right]_0^3 = 3(4c - c^2) \Rightarrow 18 - 9 = 3(4c - c^2) \quad ①$$

$$c^2 - 4c + 3 = 0, \quad (c-3)(c-1) = 0 \quad ①$$

$$c = 3 \notin (0,3), \quad c = 1 \in (0,3)$$

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③

$$f'(x) = \frac{\sin(\sqrt{x})}{\sqrt{x-1}} \cdot \frac{1}{2\sqrt{x}} - \frac{\sin(\ln x)}{\sqrt{(\ln x)^2 - 1}} \cdot \frac{1}{x}$$

① ①

$= x - 2\ln|x+1| + c$ (1)

(5)

$$\int \frac{e^{6\ln x}}{x^6}\,dx = \int \frac{e^{\ln x^6}}{x^6}\,dx$$ (2)

$$= \int dx = x + c$$

(6)

$$\int x^2\, 4^{x^3-1}\,dx$$ (2)

$$= \frac{1}{3\ln 4}\, 4^{x^3-1} + c$$

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(2)

السؤال الثاني

(1)

$$\frac{dy}{dx} = 2x\sin^{-1}(e^x) + \frac{x^2 e^x}{\sqrt{1-e^{2x}}}$$

(2)

$$y = \ln e^{3x} + \ln(\tan(x^2)) - \frac{1}{5}\ln x$$ (1)

$$y = 3x + \ln(\tan(x^2)) - \frac{1}{5}\ln x$$ (1)

$$y' = 3 + \frac{2x\sec^2(x^2)}{\tan(x^2)} - \frac{1}{5}\cdot\frac{1}{x}$$ (1)

السؤال الثالث

(1)

$$\int \left(\frac{x^3}{x^{\frac{1}{3}}} - \frac{1}{x^{\frac{1}{3}}}\right) dx$$ ($\frac{1}{2}$)

$$= \int \left(x^{\frac{8}{3}} - x^{-\frac{1}{3}}\right) dx = 3\frac{x^{\frac{11}{3}}}{11} - 3\frac{x^{\frac{2}{3}}}{2} + c$$ ($\frac{1}{2}$) (1)

(2)

$$= -\frac{1}{4}\,\frac{(2-x^4)^8}{8} + c$$ (2)

(3)

$$= \tan(x^2) + c$$ (2)

(4)

$$\int \frac{x+1-1-1}{x+1}\,dx = \int \frac{x+1}{x+1}\,dx - 2\int \frac{1}{x+1}\,dx$$ (1)

$$= \int dx - 2\int \frac{1}{x+1}\,dx$$