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الاختبار الفصلي الثاني لمقرر 111 ريض	جامعة الملك سعود – كلية العلوم- قسم الرياضيات
الفصل الأول 1436 / 1437 هـ الزمن: ساعة ونصف الدرجة: 25	الإسم / .... الرقم الجامعي / .... أستاذ المقرر / ....

ملاحظات : 1. عدد الورقات أربعة و ورقة مسودة 2. ممنوع استخدام الآلة الحاسبة

السؤال الأول : احسب $\frac{dy}{dx}$ فيما يلي:

(أ) $y = \cosh^2 x + \sinh^{-1}(x^2)$ (درجتان)

$$\frac{dy}{dx} = 2\cosh x \sinh x + \frac{2x}{\sqrt{1+x^4}}$$

1 1

(ب) $y = \tanh^{-1}(\sqrt{x})$ (درجتان)

$$\frac{dy}{dx} = \frac{1}{1-x} \cdot \frac{1}{2\sqrt{x}}$$

1 1

السؤال الثاني: احسب التكاملات التالية :

(1) $\int \frac{dx}{\sqrt{1-e^{2x}}}$ (درجتان)

نضع $u = e^x$ فإن $du = e^x dx$ إذن $dx = \frac{du}{u}$

$du = u\,dx$

(1)

$$\int \frac{dx}{\sqrt{1-e^{2x}}} = \int \frac{du}{u\sqrt{1-u^2}} = -\operatorname{sech}^{-1}(u) + c$$

$$= -\operatorname{sech}^{-1}(e^x) + c, \quad c \in \mathbb{R}$$

(1)

(2) $\int \frac{dx}{\sqrt{x^2+2x+5}}$ (درجتان)

(1) $x^2+2x+5 = (x+1)^2 + 2^2$

$$\int \frac{dx}{\sqrt{x^2+2x+5}} = \int \frac{dx}{\sqrt{2^2+(x+1)^2}} = \sinh^{-1}\left(\frac{x+1}{2}\right) + c, \quad c \in \mathbb{R}$$

(1)

(3) $\int_1^e \ln x \, dx$ (درجتان)

$u(x) = \ln x \quad \Rightarrow \quad u'(x) = \frac{1}{x}$

(1) $v'(x) = 1 \quad \Rightarrow \quad v(x) = x$

$$\int_1^e \ln x \, dx = [x \ln x]_1^e - \int_1^e 1 \, dx$$

$$= e \ln e - [x]_1^e = e - (e-1) = 1$$

(1)

$\int \sin^3 x \cos^7 x \, dx$ (4) (درجتان)

الطريقة الأولى:

$I = \int \sin^3 \cos^7 x \, dx = \int \sin^2 x \cos^7 x \sin x \, dx$

$= \int (1-\cos^2 x) \cos^7 x \sin x \, dx$ (1)

$u = \cos x$

$du = -\sin x \, dx$

$I = -\int (1-u^2) u^7 \, du = -\int (u^7 - u^9) \, du$

$= -\left[\frac{u^8}{8} - \frac{u^{10}}{10}\right] + c = \frac{\cos^{10} x}{10} - \frac{\cos^8 x}{8} + c$ (1)

طريقة ثانية:

$I = \int \sin^3 x \cos^7 x \, dx = \int \sin^3 x (\cos^2 x)^3 \cos x \, dx$

(1) $= \int \sin^3 x (1-\sin^2 x)^3 \cos x \, dx$

$u = \sin x$

$du = \cos x \, dx$

$I = \int u^3 (1-u^2)^3 \, du = \int u^3 [1 - 3u^2 + 3u^4 - u^6] \, du$

$= \int (u^3 - 3u^5 + 3u^7 - u^9) \, du$

(1) $= \frac{u^4}{4} - 3\frac{u^6}{6} + 3\frac{u^8}{8} - \frac{u^{10}}{10} + c$

$\int \sec^4 x \, dx$ (5) (درجتان)

$\int \sec^4 x \, dx = \int \sec^2 x \sec^2 x \, dx$

(1) $= \int (1 + \tan^2 x) \sec^2 x \, dx$

$u = \tan x$

$du = \sec^2 x \, dx$

$= \int (1 + u^2) \, du = u + \frac{u^3}{3} + c$

$\int \sec^4 x \, dx = \tan x + \frac{\tan^3 x}{3} + c, \; c \in \mathbb{R}$ (1)

$\int \sin(3x)\cos(2x) \, dx$ (6) مع العلم أن $2\sin a \cos b = \sin(a-b) + \sin(a+b)$ (درجتان)

(1) $\sin 3x \cos 2x = \frac{1}{2}[\sin x + \sin 5x]$

$\int \sin 3x \cos 2x \, dx = \frac{1}{2} \int [\sin x + \sin 5x] \, dx$

$= -\frac{1}{2}\left[\cos x + \frac{\cos 5x}{5}\right] + c$ (1)

$\int (1-x^2)^{3/2} \, dx$ (7) (3 درجات)

نضع $x = \sin\theta$ فإن $dx = \cos\theta \, d\theta$, $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

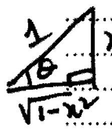
$\sqrt{1-x^2} = \sqrt{1-\sin^2\theta} = \cos\theta$

(1) $\int (1-x^2)^{3/2} \, dx = \int \cos^4\theta \, d\theta = \int \left(\frac{1+\cos 2\theta}{2}\right)^2 d\theta$

$= \frac{1}{4} \int [1 + 2\cos 2\theta + \cos^2 2\theta] \, d\theta$

(1) $= \frac{1}{4}\left[\theta + 2\frac{\sin 2\theta}{2} + \int \frac{1+\cos 4\theta}{2} \, d\theta\right]$

$$\int (1-x^2)^{3/2}\,dx = \frac{1}{4}\left[\theta + 2\sin\theta\cos\theta + \frac{\theta}{2} + \frac{\sin 4\theta}{8}\right] + c$$



$$= \frac{3}{8}\theta + \frac{1}{2}\sin\theta\cos\theta + \frac{1}{32}\sin 4\theta + C$$

$$= \frac{3}{8}\sin^{-1}x + \frac{1}{2}x\sqrt{1-x^2} + \frac{1}{32}\left[4x\sqrt{1-x^2}\left(2(1-x^2)-1\right)\right] + c$$

(1)

$\sin 4\theta = 2\sin 2\theta\cos 2\theta = 4\sin\theta\cos\theta\,[2\cos^2\theta - 1]$

(3 درجات) $\int \frac{dx}{(x-1)(x^2+1)}$ (8)

(0,5) $x \neq 1, \quad f(x) = \frac{1}{(x-1)(x^2+1)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+1}$

$A = \lim_{x\to 1}(x-1)f(x) = \lim_{x\to 1}\frac{1}{x^2+1} = 1/2$

(1,5) $0 = \lim_{x\to\infty} x f(x) = A + B \Rightarrow B = -1/2$

$f(0) = -1 = -1/2 + C \Rightarrow C = -1/2$

$x \neq 1, \quad f(x) = \frac{1}{(x-1)(x^2+1)} = \frac{1}{2}\left[\frac{1}{x-1} - \frac{x+1}{x^2+1}\right]$

$$\int \frac{dx}{(x-1)(x^2+1)} = \frac{1}{2}\left[\int\frac{dx}{x-1} - \frac{1}{2}\int\frac{2x}{x^2+1}dx - \int\frac{dx}{1+x^2}\right]$$

$$= \frac{1}{2}\ln|x-1| - \frac{1}{4}\ln(x^2+1) - \frac{1}{2}\tan^{-1}x + c, \quad c\in\mathbb{R}$$

(1)

(3 درجات) $\int \frac{dx}{1+\sin x+\cos x}$ (9)

نضع $u = \tan(x/2)$ فإنّ $dx = \frac{2\,du}{1+u^2}$

(1) $\cos x = \frac{1-u^2}{1+u^2}$

$\sin x = \frac{2u}{1+u^2}$

$$\int \frac{dx}{1+\sin x+\cos x} = \int \frac{\frac{2du}{1+u^2}}{1+\frac{2u}{1+u^2}+\frac{1-u^2}{1+u^2}}$$

(1)

$$= 2\int \frac{du}{(1+u^2)+2u+1-u^2} = \int \frac{du}{1+u}$$

$$= \ln|1+u| + c$$

(1) $= \ln|1+\tan(x/2)| + c, \quad c\in\mathbb{R}$