King Saud University
College of Sciences
Department of Mathematics

Second Mid Term Exam, S2
1439/1440
M 380 – Stochastic Processes
Time: 90 minutes

Answer the following questions:

Q1: [4+4]

a) For the Markov process $\{X_t\}$, $t=0,1,2,\ldots,n$ with states $i_0, i_1, i_2, \ldots, i_{n-1}, i_n$

Prove that: $\Pr\{X_0 = i_0, X_1 = i_1, X_2 = i_2, \ldots, X_n = i_n\} = p_{i_0} P_{i_0 i_1} P_{i_1 i_2} \ldots P_{i_{n-1} i_n}$ where $p_{i_0} = \text{pr}\{X_0 = i_0\}$

b) A Markov chain $X_0, X_1, X_2, \ldots$ has the transition probability matrix

$$\mathbf{P} = \begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \left\| \begin{array}{ccc} 0.2 & 0.3 & 0.5 \\ 0.4 & 0.2 & 0.4 \\ 0.5 & 0.3 & 0.2 \end{array} \right\| \end{array}$$

and initial distribution $p_0 = 0.5$ and $p_1 = 0.5$ Determine the probabilities

$\text{pr}\{X_0 = 1, X_1 = 1, X_2 = 0\}$ and $\text{pr}\{X_1 = 1, X_2 = 1, X_3 = 0\}$

Q2: [5+4]

a) Consider a spare parts inventory model in which either 0, 1, or 2 repair parts are demanded in any period, with $\Pr\{\xi_n = 0\} = 0.3$, $\Pr\{\xi_n = 1\} = 0.2$, $\Pr\{\xi_n = 2\} = 0.5$ and suppose $s=0$ and $S=3$. Determine the transition probability matrix for the Markov chain $\{X_n\}$, where X_n is defined to be the quantity on hand at the end of period n.

b) Let $\{X_n\}$ be a Markov chain for daily weather with two states 0, 1 (0 for dry day and 1 for rainy day) has the transition probability matrix

$$P = \begin{array}{c} \\ 0 \\ 1 \end{array} \begin{array}{c} \begin{array}{cc} 0 & 1 \end{array} \\ \left\| \begin{array}{cc} 0.7 & 0.3 \\ 0.4 & 0.6 \end{array} \right\| \end{array}$$

What's the probability for the weather to be dry today and rainy on the coming two days?

Q3: [8]

A Markov chain $X_0, X_1, X_2, \ldots$ has the transition probability matrix

$$\mathbf{P} = \begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \left\| \begin{array}{ccc} 0.5 & 0.2 & 0.3 \\ 0.5 & 0.1 & 0.4 \\ 0.3 & 0.2 & 0.5 \end{array} \right\| \end{array}$$

Every period that the process spends in state 0 incurs a cost \$4. Every period that the process spends in state 1 incurs a cost of \$7. Every period that the process spends in state 2 incurs a cost of \$5. What is the long run mean cost per period associated with this Markov chain?
