King Saud University
College of Sciences
Department of Mathematics

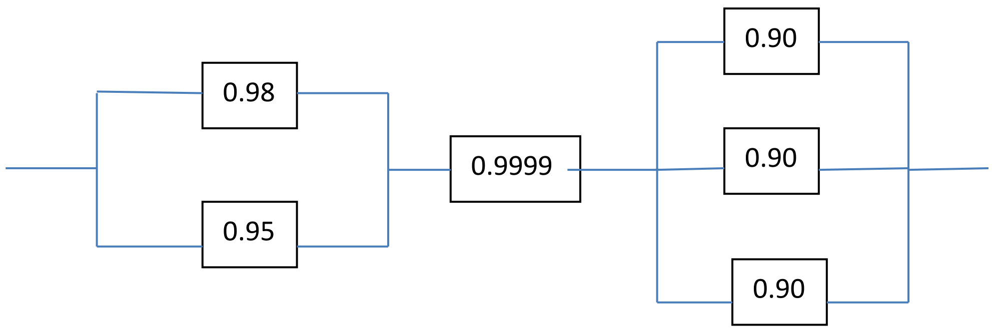
Mid-Term Exam, S1 1441
M 507 – Advanced Operation Research
Time: 90 minutes

Answer the following questions:

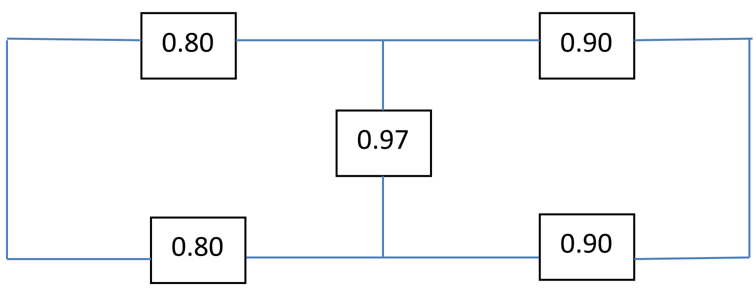
Q1: [4+6]

Compute the system reliability for the following configuration diagram where each component has the indicated reliability

a)



b)



Q2: [8]

An oil drilling company drills at a large number of locations in search of oil. The probability of success at any location is 0.25 and the locations may be regarded as independent.

a) What is the probability that the driller will experience 1 success if 10 locations are drilled?

b) The driller feels that he will go bankrupt if he drills 10 times before experiencing his first success. What is the probability that he will go bankrupt?

Q3: [12]

The life of a product follows a Weibull distribution with a shape parameter of 1.5 and a scale parameter of 100 hours.

i) What is the probability that the item fails before achieving a life of $x=34$ hours?

ii) Compute the instantaneous failure rate at its characteristic value and the average failure rate over the time interval $(80,100)$.

iii) Compute the mode, the tenth percentile, the median, the MTTF and the variance for this distribution.

Model Answer

Q1: [4+6]

a)

$$R_{sys} = [1-(1-0.98)(1-0.95)](0.9999)[1-(1-0.9)^3]$$
$$= 0.9979$$

b)

We use the decomposition method and we take the component 3 of reliability 0,97 as a pivot element.

$$R^+ = [1-(1-0.8)^2][1-(1-0.9)^2]$$
$$=0.9504$$
$$R^- = 1-(1-0.8\times 0.9)(1-0.8\times 0.9)$$
$$=0.9216$$
$$\therefore\ R_{sys} = R_3R^+ + (1-R_3)R^-$$
$$=0.97\times 0.9504 + 0.03\times 0.9216$$
$$=0.9495$$

Q2: [4+4]

a) This implies that n=10, p=0.25 and X=1

$$\therefore\ p(x=1)=\binom{10}{1}p^1q^9$$
$$=10\times 0.25\times 0.75^9$$
$$=0.1877$$

b) The probability that he will go bankrupt is given by

$$p(x=0)=\binom{10}{0}p^0q^{10}$$
$$=0.25^0\times 0.75^{10}$$
$$=0.0563$$

Q3: [12]

i)

$$\Pr(X < 34) = F(34)$$
$$=1- R(34)$$
$$=1-\exp[-(\tfrac{t}{\eta})^{\beta}]$$
$$=1-e^{-(\frac{34}{100})^{1.5}}$$
$$=0.1798$$

ii) The instantaneous failure rate is given by

$$\lambda(t) = \frac{\beta}{\eta^{\beta}} t^{\beta-1}$$

$$\lambda(100) = \frac{1.5}{100^{1.5}} \times 100^{0.5}$$
$$= \frac{1.5}{100}$$
$$=0.015$$

And the average failure rate is

$$\bar{\lambda} = \frac{t_2^{\beta} - t_1^{\beta}}{\eta^{\beta}(t_2 - t_1)}$$
$$= \frac{100^{1.5} - 80^{1.5}}{100^{1.5}(100-80)}$$
$$=0.0142$$

iii) The Mode is given by

$$x_m = \eta\left(\frac{\beta-1}{\beta}\right)^{1/\beta}$$
$$=100\left(\frac{0.5}{1.5}\right)^{1/1.5}$$
$$=48.075$$

The tenth percentile is

$$x_p = \left(\ln\left(\frac{1}{1-p} \right) \right)^{1/\beta} .\eta$$

$$= \left(\ln\left(\frac{1}{0.9} \right) \right)^{1/1.5} \times 100$$

$$=22.3076$$

Also, the median is

$$x_{0.50} = \left(\ln\left(\frac{1}{1-0.50} \right) \right)^{1/1.5} \times 100$$

$$= (\ln 2)^{2/3} \times 100$$

$$=78.32$$

The MTTF for 2p Weibull is given by

$$\mu = \eta \, \Gamma\left(\frac{1}{\beta} + 1 \right)$$

$$= \eta \, B_1$$

$$=100 \times 0.9027$$

$$=90.27$$

The variance is

$$\sigma^2 = \eta^2 [B_2 - B_1^2]$$

$$=10000 \times 0.3757$$

$$=3757$$