جامعة الملك سعود
King Saud University

Basics of Engineering Measurements

(AGE 2340)

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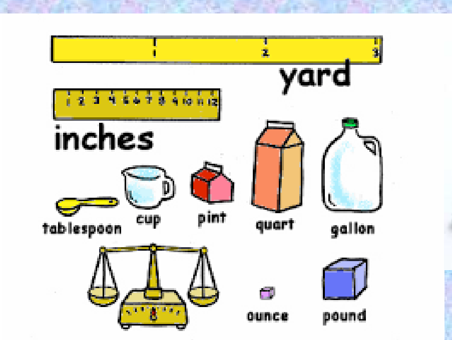
Chapter 2:

Calibration & Uncertainty Analysis

Basic Terminology of Measurement

- **Measurement**

The International Vocabulary of Basic and General Terms in Metrology , using International Organization for Standardization (ISO) norms, has defined measurement as "a set of operations having the object of determining the value of a quantity". In other words, a measurement is the evaluation of a quantity made after comparing it to a quantity of the same type which we use as a "unit".



Basic Terminology of Measurement

- **Metrology**

the science and "grammar" of measurement is defined as “the field of knowledge concerned with measurement”. Standardized measurement units mean that scientific and economic figures can be understood, reproduced, and converted with a high degree of certitude.

Basic Terminology of Measurement

- **Instrumentation**

refers to a group of permanent systems which help us measure objects. In this sense, instruments and systems of measurement constitute the "tools" of measurement and metrology.

Basic Terminology of Measurement

- **Load Effects**

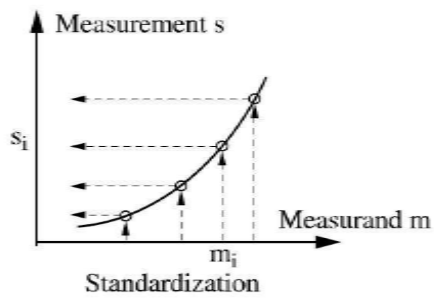
✓ **measurement operations may require connection or without contact.**

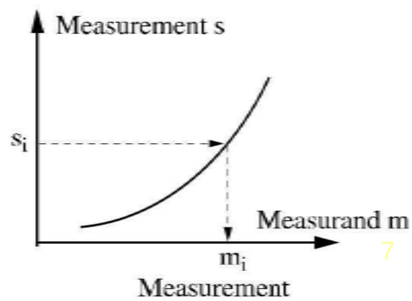
✓ **This linking of an instrument to an object or site of investigation means that a transfer of energy and/or information termed "a load effect" takes place.**

✓ **An example of this is shown by the insertion of a measuring probe into a cup of tea which takes some heat from the tea, leading to a difference between the "true" value and the value to be measured.**

Calibration

- **The relationship between the value of the input to the measurement system and the system's indicated output value is established during calibration of the measurement system.**
- **The known value used for the calibration is called the standard.**
- **The quantity to be measured being the measurand, which we call *m,* the sensor must convert *m into an electrical variable called s. The* expression *s = F(m) is established by* calibration*. By using a standard* or unit of measurement, we discover for these values of *m (m1, m2 … mi) electrical signals sent by the sensor (s1, s2 ... si) and we trace the* curve *s(m), called the sensor calibration curve.***





Accuracy & Precision

- **Accuracy of a system can be estimated during calibration. If the input value of calibration is known exactly, then it can called the *true value. The accuracy of a measurement system refers to its* ability to indicate a true value exactly.**
- **Accuracy : It is the ability of instrument to tell the truth**
- **Accuracy is related to *absolute error, ε:***

  ε = true value – indicated value

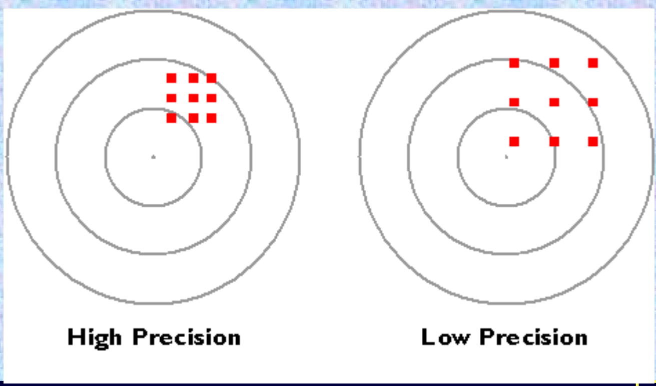
  from which the percent accuracy is found by :

$$A = \left(1 - \frac{|\varepsilon|}{\text{true value}}\right) x100$$

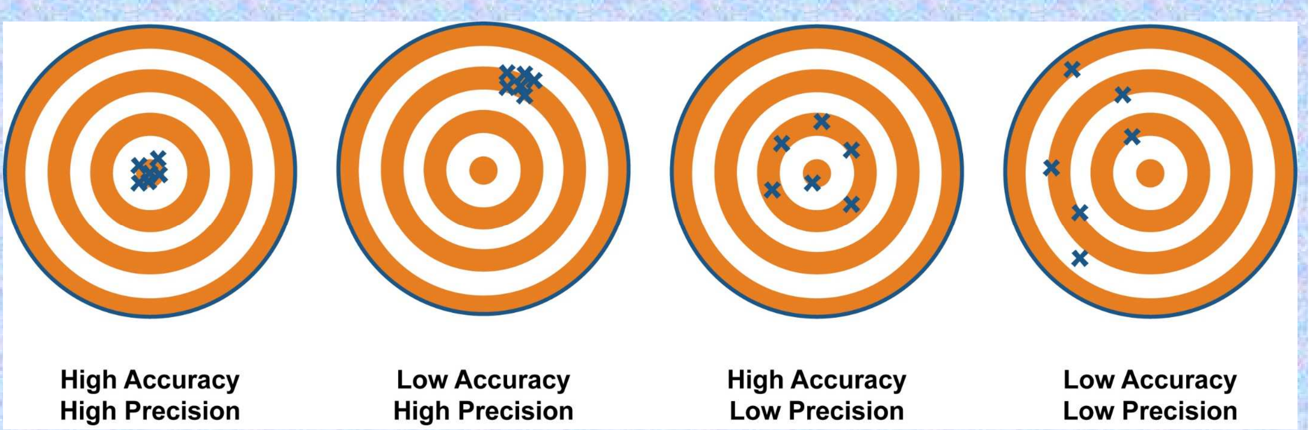
Accuracy & Precision

- **Precision:** or repeatability of a measuring system refers to the ability of the system to indicate a particular value upon repeated but independent applications of a specific value input. Precision of a measurement describes the units used to measure something.

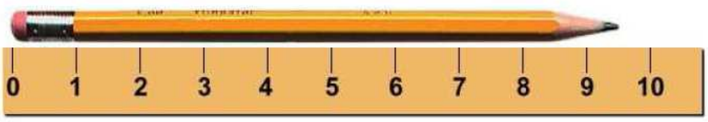
- **Precision :** It is the ability of the instrument to give the same output for the same input under the same conditions



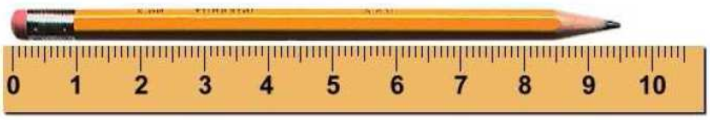
Accuracy & Precision



Precision Example: How long is the pencil?



The best you can say is 'about 9 centimetres'



The best you can say is 'about 9.5 centimetres'. This second measurement is more precise, because you used a smaller unit to measure with.

- **It is impossible to make a perfectly precise measurement.**
- **Accuracy can be improved up to but not beyond the precision of the instrument by calibration.**

Term Used in Instrument Rating

- **Resolution:** **The smallest increment of change in the measured value that can be determined from the instrument's readout scale. The resolution is often on the same order as the precision; sometimes it is smaller.**

- **Sensitivity:** **The change of an instrument's output per unit change in the measured quantity. Typically, an instrument with higher sensitivity will have also finer resolution, better precision, and higher accuracy.**

- **Range:** **The proper procedure for calibration is to apply known inputs ranging from the minimum to the maximum values for which the measurement system is to be used. These limits the operating range of the system.**

Error Classifications

➤ 1. Systematic, Fixed or Bias Errors:

- Insidious in nature, exist unnoticed unless deliberately searched.
- Repeated readings to be in error by the same amount.
- Not susceptible to statistical analysis.
  - Calibration errors
  - Certain consistently recurring human error
  - Technique error
  - Uncorrected loading error
  - Limitations of system resolution

Error Classifications

➤ 2. Precision or Random Errors:

- Distinguished by their lack of consistency. Usually (not always) follow a certain statistical distribution.
- In many instances very difficult to distinguish from bias errors.
  - Error stemming from environmental variations
  - Certain type of human error
  - Error resulting from variations in definition.

Error Classifications

➢ 3. Illegitimate Errors

Illegitimate Errors are simply mistakes on the part of experimenter.

- **Can be eliminated through the exercise of care and repetition of the measurement.**
  - **Blunders and mistakes**
  - **Computational errors**
  - **Chaotic errors.**

Ideal Distinction: *bias* versus *random* errors

Bias error is a systematic inaccuracy caused by a mechanism that we can (ideally) control. We might be able to adjust the way measurement are taken in an attempt to reduce bias errors. We can try to correct bias errors by including adjustments in our data analysis *after* the measurements are taken.

Random error is a non-repeatable inaccuracy caused by an unknown or an uncontrollable influence. Random errors introduce scatter in the measured values, and propagate through the data analysis to produce scatter in values computed from the measurements. Ideally random errors establish the limits on the precision of a measurement, not on the accuracy of a measurement.

Bias & Precision Errors

- **Bias (Systematic) Error**

✓ **is the difference between the average value in a series of repeated calibration measurements and the true value.**

✓ **Systematic error causes an offset between the mean value of the data set and its true value**

✓ **systematic error = average value - true value**

- **Precision Error (Random error)**

✓ **is a measure of the random variation found during repeated measurements.**

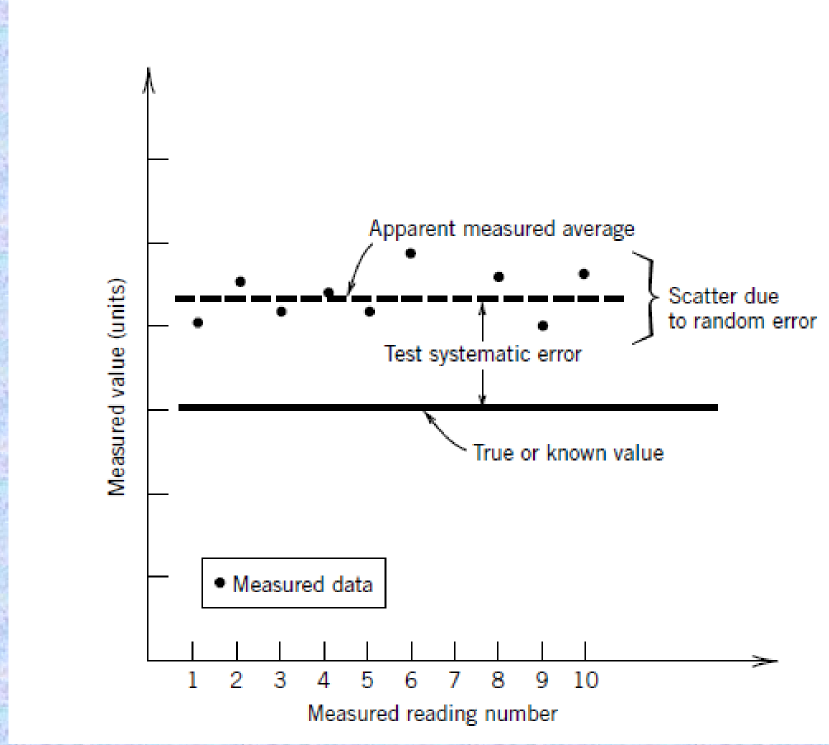
✓ **random error = reading - average of readings**

✓ **Random error causes a random variation in measured values found during repeated measurements of a variable**

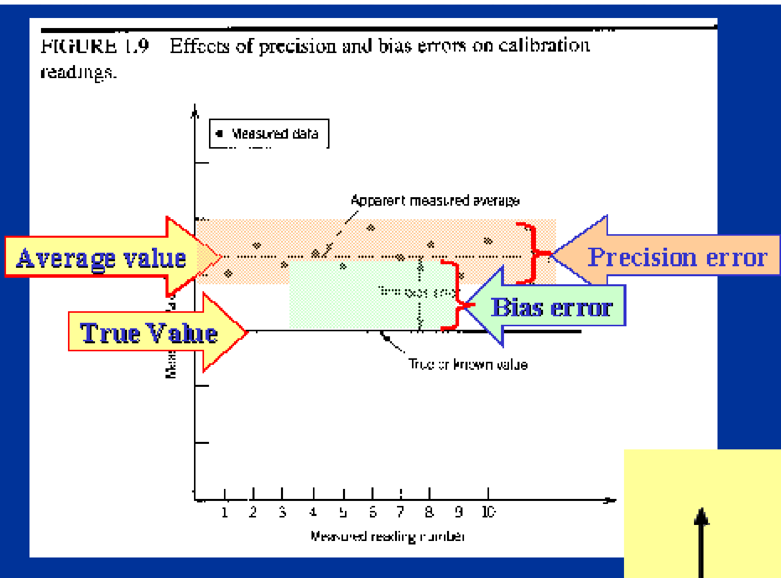
❑ **Both random and systematic errors affect a system's accuracy.**

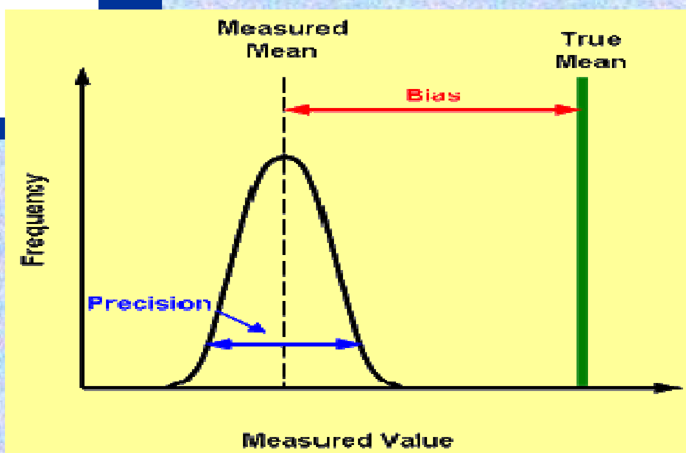
systematic error	random error
poor accuracy	poor precision
definite causes	nonspecific causes
reproducible	not reproducible

Bias & Precision Errors

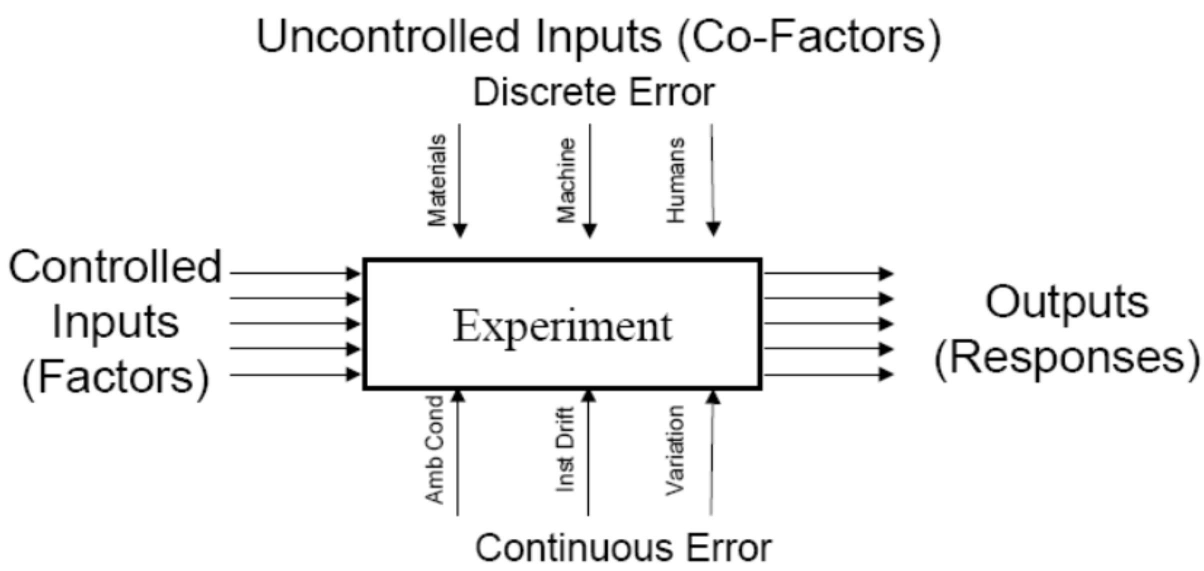


Effects of precision and bias errors on calibration readings



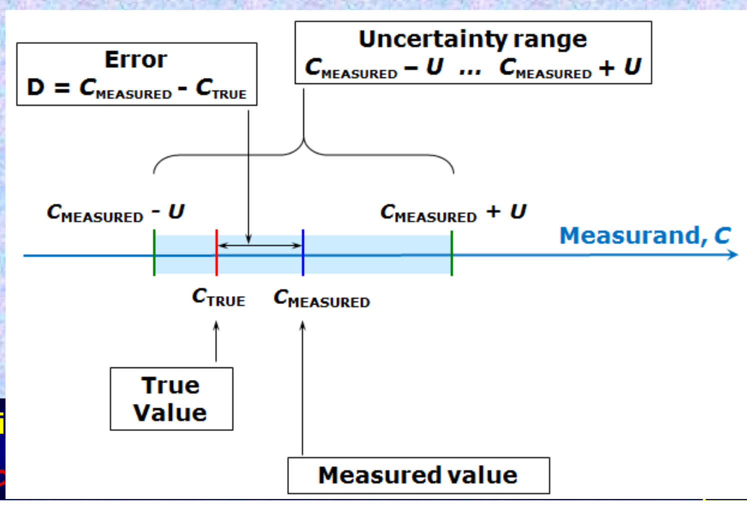


SOURCE OF ERRORS



Uncertainty

- ✓ The uncertainty is a numerical estimate of the possible range of the error in a measurement.
- ✓ In any measurement, the error is not known exactly since the true value is rarely known exactly.
- ✓ that the error is within certain bounds, a plus or minus range of the indicated reading



Simplified Error Estimation

- Cosider the calculation of electrical power, $P = EI$

$$E = 100\ \mathrm{V} \pm 5\ \mathrm{V} \qquad I = 10\ \mathrm{A} \pm 0.1\ \mathrm{A}$$

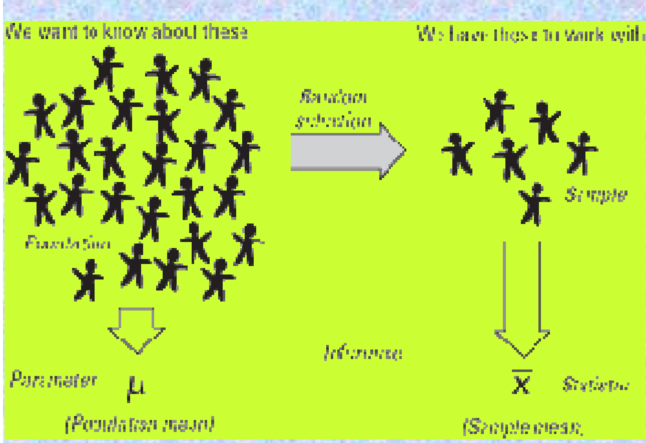
- The nominal value of power is $100 \times 10 = 1000$ W, &

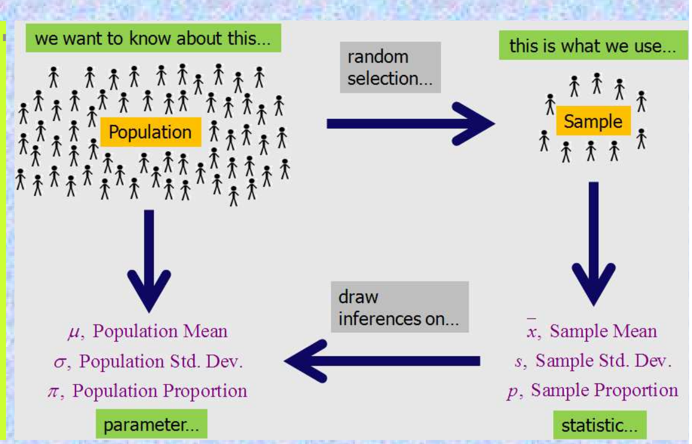
$$\begin{aligned} P_{max} &= (100+5)(10+0.1) = 1060.5\ \mathrm{W} \\ P_{min} &= (100-5)(10-0.1) = 940.5\ \mathrm{W} \end{aligned}$$

- The uncertainty in the power is $+6.05\%$, -5.95%.
- It is quite unlikely that the power would be in error by these amount. Hence, comprehensive uncertainty analysis is required.

Statistics

Population & Sample





Notation

- Σ denotes the addition of a set of values
- x is the variable usually used to represent the individual data values
- n represents the number of data values in a sample
- N represents the number of data values in a population

Definitions

Mean (Average): the number obtained by adding the values and dividing the total by the number of values.

Median: the middle value when the original data values are arranged in order of increasing (or decreasing) magnitude.

Variance: It is the expectation of the squared deviation of a random variable from its mean

Standard Deviation: a measure of variation of the scores about the mean (average deviation from the mean)

Sample and Population Mean

Sample Mean	Population Mean
$\bar{x} = \dfrac{\Sigma x}{n}$	$\mu = \dfrac{\Sigma x}{N}$

where Σx is sum of all data values

N is number of data items in population

n is number of data items in sample

Sample and Population Variance

$$s^2 = \frac{\Sigma (x - \bar{x})^2}{n - 1}$$ Sample Variance

$$\sigma^2 = \frac{\Sigma (x - \mu)^2}{N}$$ Population Variance

Sample and Population Standard Deviations

$$s_x = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n - 1}}$$

$n =$ The number of data points

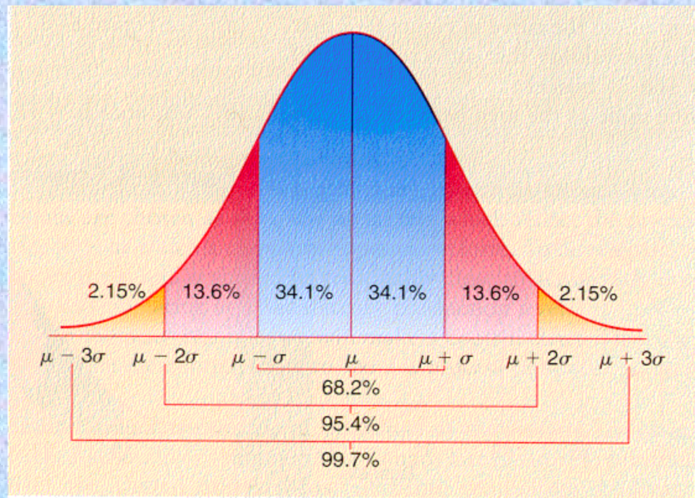
$\bar{x} =$ The mean of the x_i

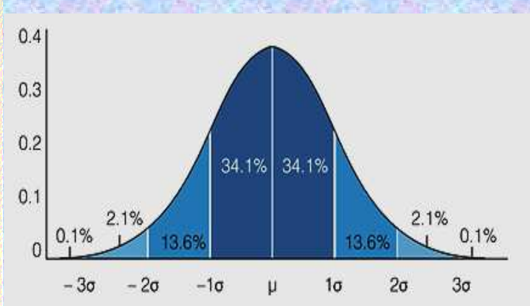
$x_i =$ Each of the values of the data

$$\sigma = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (x_i - \mu)^2},$$

Normal Distribution

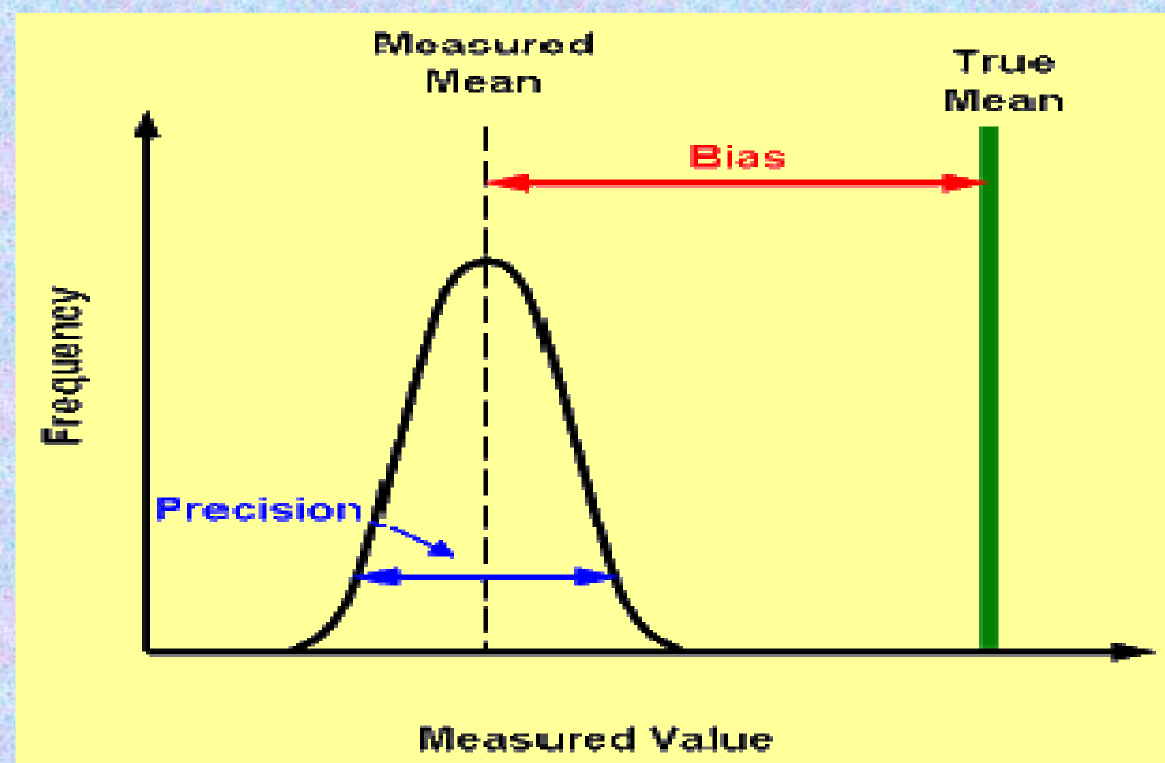
The normal distribution is a continuous probability distribution that has a bell-shaped probability density function, known as the Gaussian function, or informally, the bell curve.





Uncertainty Analysis

- The estimate of the error is called the uncertainty.
  - It includes both bias and precision errors.
    - We need to identify all the potential significant errors for the instrument(s).
  - All measurements should be given in three parts
    - Mean value
    - Uncertainty
    - Confidence Interval on which that uncertainty is based (*typically 95% C.I.*)
- Uncertainty can be expressed in either absolute terms (i.e., 5 Volts ±0.5 Volts)
  or in
  percentage terms (i.e., 5 Volts ±10%)
  (relative uncertainty = DV / V x 100)
  - We will use a 95 % confidence interval throughout this course



Calculation of bias Uncertainty

- Manufacturers’ Specifications
  - If you can’t do better, you may take it from the manufacturer’s specs.
  - Accuracy - %FS, %reading, offset, or some combination (e.g., 0.1% reading + 0.15 counts)
    - Unless you can identify otherwise, assume that these are at a 95% confidence interval
- Independent Calibration
  - May be deduced from the calibration process

Calculation of precision Uncertainty

- Use Statistics to Estimate Random Uncertainty
- **Mean:** the sum of measurement values divided by the number of measurements.

$$\bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i$$

- **Deviation:** the difference between a single result and the mean of many results. $d_i = x_i - \bar{x}$
- **Standard Deviation:** the smaller standard deviation is the more precise data
  - Large sample size

$$\sigma = \left[\frac{1}{n}\sum (x_i - \bar{x})^2\right]^{\frac{1}{2}}$$

  - Small sample size (n<30)
    Slightly larger value

$$\sigma_s = \left[\frac{1}{n-1}\sum (x_i - \bar{x})^2\right]^{\frac{1}{2}}$$

Calculation of precision Uncertainty

The precision uncertainty has to be calculated by estimating mean of the sample reading and standard deviation of the sample.
Let x_m be the mean and S_x be the standard deviation of the sample for which n repetitions were made.

Then the precision uncertainty will be given by : $U_p = \pm\, t_{\frac{\alpha}{2},\vartheta} \frac{s_x}{\sqrt{n}}$

Each of the individual measurement variables (X_1, X_2,…,X_K) is subject to Several precision errors.

The bias limits for each of these elemental sources are combined in some manner to obtain the overall bias limit (U_{P1}, U_{P2},…, U_{PK}) for each variable.

Calculation of precision Uncertainty

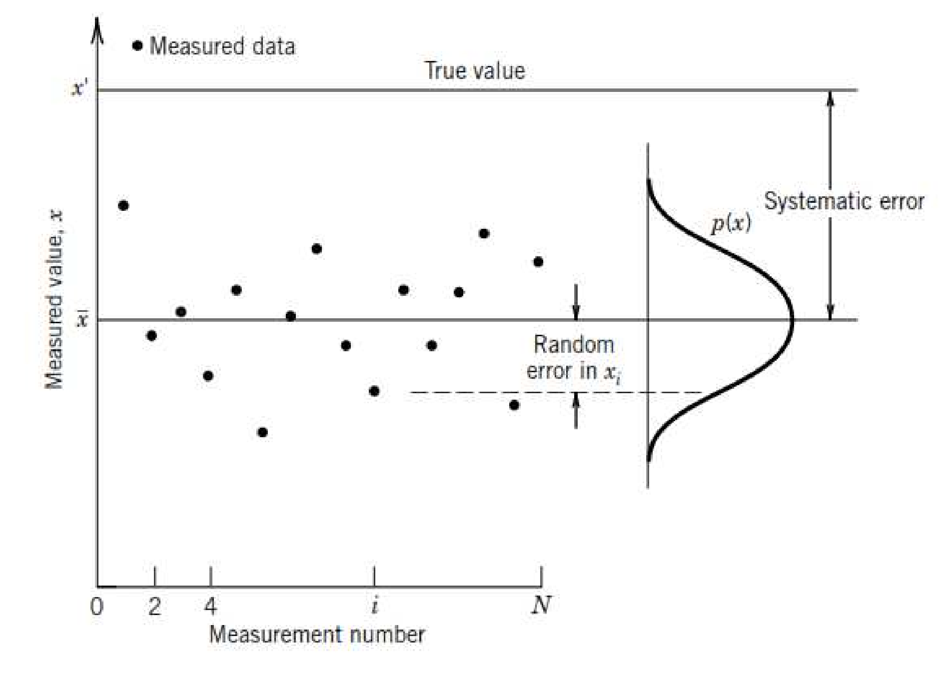
How to calculate : $U_p = \pm\, t_{\frac{\alpha}{2},\vartheta} \frac{s_x}{\sqrt{n}}$

By using T – distribution:

n = number of samples
ν = n – 1 = degree of freedom
α = 1 – c , c: is the confidence

$$s_x = \sigma_s = \left[\frac{1}{n-1}\sum(x_i - \bar{x})^2\right]^{\frac{1}{2}}$$

$$\overline{X} \pm t_{\alpha/2,\, n-1} \frac{s}{\sqrt{n}}$$



Distribution of errors on repeated measurements.

Student t-distribution (small sample sizes)

- **The t-distribution was formulated by W.S. Gosset, a scientist in the Guinness brewery in Ireland, who published his formulation in 1908 under the pen name (pseudonym) "Student."**
- **The t-distribution looks very much like the Gaussian distribution, bell shaped, symmetric and centered about the mean. The primary difference is that it has stronger tails, indicating a lower probability of being within an interval. The variability depends on the sample size, n.**
- **With a confidence interval of c%**

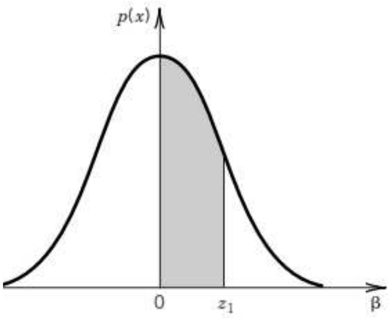
$$\bar{x} - t_{\alpha/2,\nu}\frac{\sigma_s}{\sqrt{n}} < X < \bar{x} + t_{\alpha/2,\nu}\frac{\sigma_s}{\sqrt{n}}$$

  - **Where α=1-c and ν=n-1 (Degrees of Freedom)**

Don't apply blindly - you may have better information about the population than you think.

Table 4.3 Probability Values for Normal Error Function: One-Sided Integral Solutions for $p(z_1) = \frac{1}{(2\pi)^{1/2}} \int_0^{z_1} e^{-\beta^2/2} d\beta$

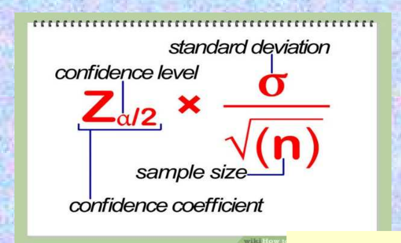
$z_1 = \frac{x_1 - x'}{\sigma}$	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1809	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2794	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4292	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4758	0.4761	0.4767
2.0	0.4772	0.4778	0.4803	0.4788	0.4793	0.4799	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.49865	0.4987	0.4987	0.4988	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990



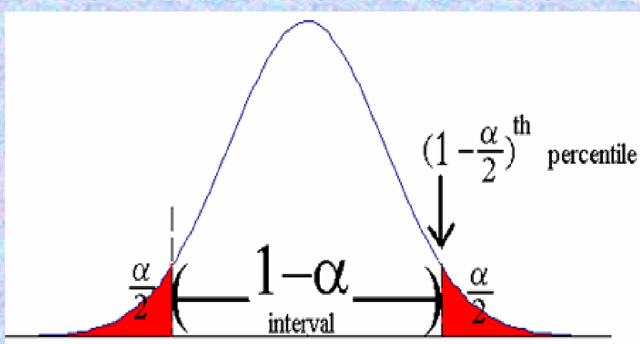
Confidence Interval

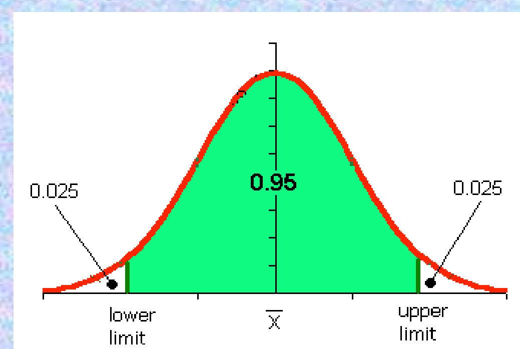
Confidence Interval

A **confidence interval** (or **interval estimate**) is a range (or an interval) of values used to estimate the true value of a population parameter. A confidence interval is sometimes abbreviated as CI.



$$\overline{X} \pm t \frac{s}{\sqrt{n}}$$





Use *Z*-distribution

If the population standard deviation is known or the sample is greater than 30.

$$\bar{X} \pm z \frac{\sigma}{\sqrt{n}}$$

Use *t*-distribution

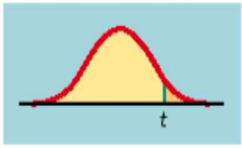
If the population standard deviation is unknown and the sample is less than 30 and normally distributed.

$$\bar{X} \pm t \frac{s}{\sqrt{n}}$$

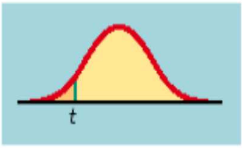
$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

One-sided (one-tailed)

$H_a: \mu > \mu_0 \Rightarrow P(T \geq t)$

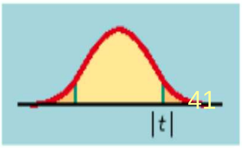


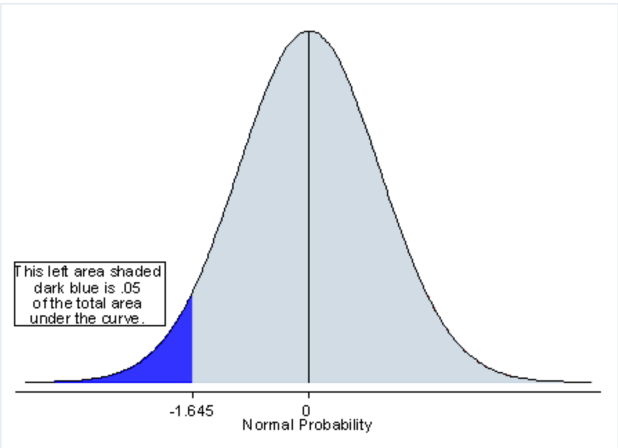
$H_a: \mu < \mu_0 \Rightarrow P(T \leq t)$

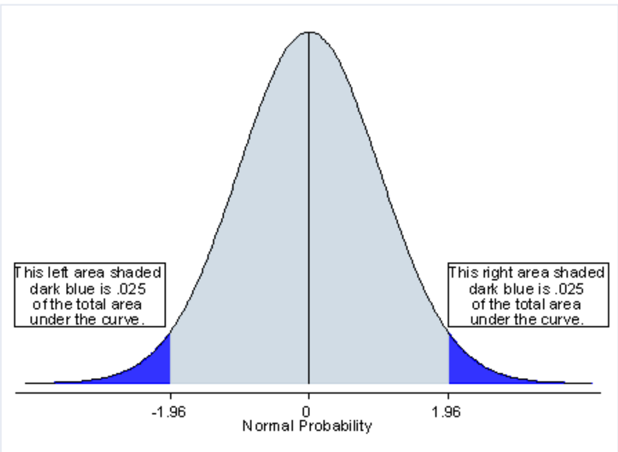


Two-sided (two-tailed)

$H_a: \mu \neq \mu_0 \Rightarrow 2P(T \geq |t|)$

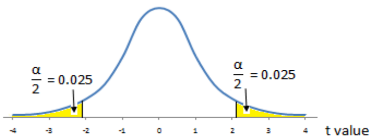






Student's t Distribution Table

For example, the t value for 18 degrees of freedom is 2.101 for 95% confidence interval (**2-Tail** α = 0.05).



	90%	95%	97.5%	99%	99.5%	99.95%	**1-Tail Confidence Level**
	80%	90%	95%	98%	99%	99.9%	**2-Tail Confidence Level**
	0.100	0.050	0.025	0.010	0.005	0.0005	**1-Tail Alpha**
df	0.20	0.10	0.05	0.02	0.01	0.001	**2-Tail Alpha**
1	3.0777	6.3138	12.7062	31.8205	63.6567	636.6192	
2	1.8856	2.9200	4.3027	6.9646	9.9248	31.5991	
3	1.6377	2.3534	3.1824	4.5407	5.8409	12.9240	
4	1.5332	2.1318	2.7764	3.7469	4.6041	8.6103	
5	1.4759	2.0150	2.5706	3.3649	4.0321	6.8688	
6	1.4398	1.9432	2.4469	3.1427	3.7074	5.9588	
7	1.4149	1.8946	2.3646	2.9980	3.4995	5.4079	
8	1.3968	1.8595	2.3060	2.8965	3.3554	5.0413	
9	1.3830	1.8331	2.2622	2.8214	3.2498	4.7809	
10	1.3722	1.8125	2.2281	2.7638	3.1693	4.5869	
11	1.3634	1.7959	2.2010	2.7181	3.1058	4.4370	
12	1.3562	1.7823	2.1788	2.6810	3.0545	4.3178	
13	1.3502	1.7709	2.1604	2.6503	3.0123	4.2208	
14	1.3450	1.7613	2.1448	2.6245	2.9768	4.1405	
15	1.3406	1.7531	2.1314	2.6025	2.9467	4.0728	
16	1.3368	1.7459	2.1199	2.5835	2.9208	4.0150	
17	1.3334	1.7396	2.1098	2.5669	2.8982	3.9651	
18	1.3304	1.7341	2.1009	2.5524	2.8784	3.9216	
19	1.3277	1.7291	2.0930	2.5395	2.8609	3.8834	
20	1.3253	1.7247	2.0860	2.5280	2.8453	3.8495	
21	1.3232	1.7207	2.0796	2.5176	2.8314	3.8193	
22	1.3212	1.7171	2.0739	2.5083	2.8188	3.7921	
23	1.3195	1.7139	2.0687	2.4999	2.8073	3.7676	
24	1.3178	1.7109	2.0639	2.4922	2.7969	3.7454	
25	1.3163	1.7081	2.0595	2.4851	2.7874	3.7251	
26	1.3150	1.7056	2.0555	2.4786	2.7787	3.7066	
27	1.3137	1.7033	2.0518	2.4727	2.7707	3.6896	
28	1.3125	1.7011	2.0484	2.4671	2.7633	3.6739	
29	1.3114	1.6991	2.0452	2.4620	2.7564	3.6594	
30	1.3104	1.6973	2.0423	2.4573	2.7500	3.6460	

Student t-distribution

- Example: t-distribution
- Sample data
  - n = 21
  - Degrees of Freedom = n-1 = 20
- Desire 95% Confidence Interval
  - $\alpha = 1 - c = 0.05$
  - $\alpha/2 = 0.025$
- Student t-distribution chart
  - t=2.086

Reading Number	Volts, mv
1	5.30
2	5.73
3	6.77
4	5.26
5	4.33
6	5.45
7	6.09
8	5.64
9	5.81
10	5.75
11	5.42
12	5.31
13	5.86
14	5.70
15	4.91
16	6.02
17	6.25
18	4.99
19	5.61
20	5.81
21	5.60
Mean	5.60
Standard dev.	0.51
Variance	0.26

Propagation of Uncertainty

- Uncertainty is based on a careful specification of the uncertainties in the various primary experimental measurements.

- If result $R = R(x_1, x_2, \cdots, x_n)$ *is a given function of independent* variables $x_1, x_2, \cdots, x_n$; *and* $w_1, w_2, \cdots, w_n$ *are the associated* uncertainties, then uncertainty of the result w_R *is given by:*

$$w_R = \sqrt{\left(\frac{\partial R}{\partial x_1} w_1\right)^2 + \left(\frac{\partial R}{\partial x_2} w_2\right)^2 + \cdots + \left(\frac{\partial R}{\partial x_n} w_n\right)^2}$$

Propagation of Uncertainty

The bias uncertainty U_B is given by manufacturer. Usually a formula for the calculation is given in the catalogue.

Each of the individual measurement variables ($X_1, X_2, \ldots, X_K$) is subject to Several bias and precision errors.

The bias error :

$$U_B = (U_{B1}, U_{B2}, \ldots, U_{Bk})$$

The precision error :

$$U_P = (U_{P1}, U_{P2}, \ldots, U_{Pk})$$

Calculation of bias Uncertainty

1- For measurement variable X this is given by:

$$U_B = (U_{B1}^2 + U_{B2}^2 + \ldots + U_{Bk}^2)^{1/2}$$

2- The next step in the procedure is to apply uncertainty analysis to determine how the bias limits $U_B = (U_{B1}, U_{B2}, \ldots, U_{Bk})$ for the individual variables propagate through the data reduction equation to form the bias limit U_B for the experimental result. The data reduction equation is taken to be of the form :

$$R = R(X_1, X_2, \ldots, X_K)$$

The bias Uncertainty is :

$$\left(\frac{U_B}{R}\right)^2 = \left(\frac{1}{R}\frac{\partial R}{\partial X_1}U_{B1}\right)^2 + \left(\frac{1}{R}\frac{\partial R}{\partial X_2}U_{B2}\right)^2 + \ldots + \left(\frac{1}{R}\frac{\partial R}{\partial X_K}U_{Bk}\right)^2$$

Calculation of precision Uncertainty

1- For measurement variable X, Then, the overall precision limit UP is given by the

$$U_P = (U_{P1}^2 + U_{P2}^2 + \ldots + U_{Pk}^2)^{1/2}$$

2- The next step in the procedure is to apply uncertainty analysis to determine how the bias limits $UP = (U_{P1}, U_{P2}, \ldots, U_{Pk})$ for the individual variables propagate through the data reduction equation. The data reduction equation is taken to be of the form :

$$R = R(X_1, X_2, \ldots, X_K)$$

the 95% precision limit for the experimental result UP is found from the uncertainty analysis expression

$$\left(\frac{U_P}{R}\right)^2 = \left(\frac{1}{R}\frac{\partial R}{\partial X_1}U_{P1}\right)^2 + \left(\frac{1}{R}\frac{\partial R}{\partial X_2}U_{P2}\right)^2 + \ldots + \left(\frac{1}{R}\frac{\partial R}{\partial X_K}U_{Pk}\right)^2$$

Overall Uncertainty

Uncertainty of the Experimental Result

In order to determine the *overall uncertainty* U_R of the experimental result, the bias and precision limits, W_R and P_R, must be combined. This is accomplished using the root-sum-square (RSS) method

$$U_R = \left(U_B^2 + U_P^2\right)^{\frac{1}{2}}$$

Uncertainty in Power Estimation

- Consider the calculation of electrical power, *P = EI:*

$$E = 100 \text{ V} \pm 5 \text{ V} \qquad I = 10 \text{ A} \pm 0.1 \text{ A}$$

$$\frac{\partial P}{\partial E} = I = 10 \text{ A} \qquad \omega_E = 5 \text{ V}$$

$$\frac{\partial P}{\partial I} = E = 100 \text{ V} \qquad \omega_I = 0.1 \text{ A}$$

$$\omega_P = \sqrt{(10 \times 5)^2 + (100 \times 0.1)^2} = \sqrt{2500 + 100} = 51.0 \text{ W} = 5.1\%$$