King Saud University	College of Sciences		Department of Mathematics
First Examination	Math 228	Semester II (1443)	Time:1H30

Exercise 1: (9 marks)

1. Find the value of the following sum $\sum_{k=1}^{10}(2k^2+1)$. (2 marks)
2. Find the average value of the function $x^3 - x + 2$ on the interval $[-1, 1]$. (2 marks)
3. Find the derivative of $F(x) = \int_{2x}^{x^2} \frac{1}{\sqrt{t^3+1}}\, dt$ (1 marks)
4. Find the area of the region bounded by the graphs of the functions $y = 2x$, $y = x$ and $-1 \leqslant x \leqslant 1$. (2 marks)
5. Find the volume of the solid obtained by rotating the region bounded by the graphs of the functions $y = e$, $y = e^x$ and y-axis about the x-axis.(Use Washer method) (2 marks)

Exercise 2: (16 marks)

Evaluate the following integrals:

1. $\int x^2 e^{2x}\, dx$. (3 marks)
2. $\int (3\sin x + 1)^4 \cos x\, dx$. (2 marks)
3. $\int \cos^2 x\, dx$. (2 marks)
4. $\int \frac{1}{(x-1)(x+2)}\, dx$. (3 marks)
5. $\int \frac{x^2}{4+x^6}\, dx$. (3 marks)
6. $\int \frac{1}{x^2\sqrt{16-x^2}}\, dx$. (3 marks)

Exercice 1 :

1)
$$\sum_{k=1}^{10}(2k^2+1) = 2\sum_{k=1}^{10}k^2 + \sum_{k=1}^{10}1.$$
$$= 2\left(\frac{10(11)(21)}{6}\right) + 10$$
$$= 2(5(11)(7)) + 10 = 780$$

2)
$$f_{av} = \frac{1}{1-(-1)}\int_{-1}^{1} x^3 - x + 2 \, dx.$$
$$= \frac{1}{2}\left[\frac{x^4}{4} - \frac{x^2}{2} + 2x\right]_{-1}^{1}$$
$$= \frac{1}{2}\left(\frac{1}{4} - \frac{1}{2} + 2 - \left(\frac{1}{4} - \frac{1}{2} - 2\right)\right) = \frac{1}{2}\times 4 = 2.$$

3)
$$F'(x) = \frac{1}{\sqrt{x^6+1}}2x - \frac{1}{\sqrt{8x^3+1}}2 = \frac{2x}{\sqrt{x^6+1}} - \frac{2}{\sqrt{8x^3+1}}.$$

4) $y = 2x$, $y = x$.

$x = 2x \Rightarrow x = 0.$

$2x - x = 0$

if $-1 \le x \le 0$, $2x \le x$; if $0 \le x \le 1$, $2x \ge x$.

$$A = \int_{-1}^{0}(x-2x)\,dx + \int_{0}^{1}(2x-x)\,dx$$
$$= \int_{-1}^{0} -x\,dx + \int_{0}^{1} x\,dx$$
$$= \left[-\frac{x^2}{2}\right]_{-1}^{0} + \left[\frac{x^2}{2}\right]_{0}^{1} = \frac{1}{2} + \frac{1}{2} = 1.$$

5)

$y = e^x$, $y = e$

$e^x = e \Rightarrow x = 1.$

y axis $x = 0.$

$\Rightarrow 0 \leq x \leq 1.$

$\Rightarrow 1 \leq e^x \leq e$

so $y = e \geq y = e^x.$

$$V = \pi \int_0^1 (e^2 - (e^x)^2)\, dx$$

$$= \pi \left[e^2 x - \frac{1}{2} \cdot e^{2x} \right]_0^1 .$$

$$= \pi \left[e^2 - \frac{1}{2} e^2 + \frac{1}{2} \right]$$

$$= \frac{1}{2} \pi (e^2 + 1).$$

<u>Exercice 2:</u>

1)

$$\int x^2 e^{2x}\, dx ,$$

$u = x^2$, $du = 2x\,dx.$

$v' = e^{2x}_{dx} \rightarrow v = \frac{1}{2} e^{2x}.$

$$= \frac{1}{2} x^2 e^{2x} - \int \frac{1}{2} e^{2x}\, 2x\, dx$$

$$= \frac{1}{2} x^2 e^{2x} - \int x e^{2x}\, dx$$

$u = x \longrightarrow du = 1\, dx.$

$v' = e^{2x} \rightarrow v = \frac{1}{2} e^{2x}.$

$$= \frac{1}{2} x^2 e^{2x} - \frac{1}{2} x e^{2x} + \frac{1}{2} \int e^{2x}\, dx.$$

$$= \frac{1}{2} x^2 e^{x} - \frac{1}{2} x e^{2x} + \frac{1}{4} e^{2x} + c.$$

2) $\int (3\sin x+1)^4 \cos x\, dx.$

$u(x) = 3\sin x + 1$, $du = \cos x\, dx.$

$= \int (3u+1)^4 du$

$= \frac{1}{3}\int 3\,(3u+1)^4 du$

$= \frac{1}{3}\,\frac{(3u+1)^5}{5} + c.$

$= \frac{(3u+1)^5}{15} + c.$

$= \frac{(3\sin x+1)^5}{15} + c.$

3) $\int \cos^2 x\, dx$

$= \int \frac{1+\cos 2x}{2} dx$

$= \frac{1}{2}\int (1+\cos 2x)\, dx$

$= \frac{1}{2}\left(x + \frac{1}{2}\sin 2x\right) + c$

$= \frac{1}{2}x + \frac{1}{4}\sin 2x + c$

4.) $\frac{1}{(x-1)(x+2)} = \frac{A}{x-1} + \frac{B}{x+2}$

$= \frac{A(x+2)+B(x-1)}{(x-1)(x+2)}$

$\Rightarrow A(x+2) + B(x-1) = 1.$

$\underline{x=1}$: $\Rightarrow 3A = 1 \Rightarrow A = \frac{1}{3}.$

$\underline{x=-2}$ $\Rightarrow -3B = 1 \Rightarrow B = -\frac{1}{3}.$

$\Rightarrow \frac{1}{(x-1)(x+2)} = \frac{1}{3(x-1)} - \frac{1}{3(x+2)}$

$\Rightarrow \int \frac{1}{(x-1)(x+2)} dx = \frac{1}{3}\int \frac{1}{x-1} dx - \frac{1}{3}\int \frac{1}{x+2} dx$

$= \frac{1}{3}\ln|x-1| - \frac{1}{3}\ln|x+2| + c$

$= \frac{1}{3}\ln\left|\frac{x-1}{x+2}\right|.$

5.) $\int \frac{x^2}{4+x^6} dx =$

$u = x^3$; $du = 3x^2 dx.$

$= \frac{1}{3}\int \frac{1}{2^2+u^2} du = \frac{1}{3}\,\frac{1}{2}\tan^{-1}\left(\frac{u}{2}\right) + c$

$= \frac{1}{6}\tan^{-1}\left(\frac{x^3}{2}\right) + c.$

6) $\int \frac{1}{x^2\sqrt{16-x^2}}\,dx$.

$x = 4\sin\theta$; $dx = 4\cos\theta\,d\theta$; $\sqrt{16-x^2} = \sqrt{16-16\sin^2\theta} = \sqrt{16(1-\sin^2\theta)}$

$= \sqrt{16\cos^2\theta} = 4\cos\theta$.

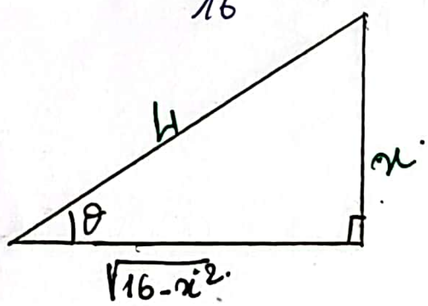
$\int \frac{1}{x^2\sqrt{16-x^2}}\,dx = \int \frac{4\cos\theta}{16\sin^2\theta \cdot 4\cos\theta}\,d\theta$.

$= \int \frac{1}{16\sin^2\theta}\,d\theta$.

$= \frac{1}{16}\int \csc^2\theta\,d\theta = -\frac{1}{16}\cot(\theta) + C$.

$x = 4\sin\theta$.

$\Rightarrow \sin\theta = \frac{x}{4}$.



$\cot(\theta) = \frac{\sqrt{16-x^2}}{x}$

$\Rightarrow \int \frac{1}{x^2\sqrt{16-x^2}}\,dx = -\frac{1}{16}\frac{\sqrt{16-x^2}}{x} + C$

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