

**Student Name:**

**ID#:**

**Answer the following problem**

**Problem 1**

For the simple linear regression model

$$Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i,$$

$\varepsilon_i, i = 1, 2, \dots, n, \text{ are independent, } \text{Var}(\varepsilon_i) = \sigma^2 \text{ and } E(\varepsilon_i) = 0$

(a) Find the least square estimators of  $\beta_0, \beta_1$

(b) Show that the sum of the residuals equal to zero.

(c) Prove that the point  $(\bar{X}, \bar{Y})$  will always fall on the fitted line.

(d) Show that the sum of total sum of square errors is the summation of the sum of square errors and the sum of the square regression errors.

## **Problem 2**

The following data represents the income (Inc) and consumption (Con ) in SR of 10 families in a certain city.

Inc	Con
8559.4	6830.4
8883.3	7148.8
9060.1	7439.2
9378.1	7804.0
9937.2	8285.1
10485.9	8819.0
11268.1	9322.7
11894.1	9826.4
12238.8	10129.9
12030.3	10088.5

Use the simple linear regression model to:

(a) Estimate the regression line and interpret the coefficients.

(b) Construct 90% confidence intervals for the model coefficients and explain the results.

(c) Test the linearity by using two different approaches.

(d) Calculate the residual at  $\text{Inc} = 11268.1$  and  $\text{con} = 9322.7$

(e) Estimate the standard deviation of the residuals.

### **Problem 3:**

A linear regression was run on a set of data. You are given only the following partial information:

Predictor	Coef	SE Coef	T
Constant	293.89	5.62	
X		0.13	-13.13

Analysis of Variance				
Source	DF	SS	MS	F
Regression				
Residual Error	5		44.21	
Total				

(a) Compute the 95% Confidence intervals for  $\beta_0$  and  $\beta_1$

(b) Give the F-statistic and test  $H_0: \beta_1 = 0$  vs  $H_1: \beta_1 \neq 0$

(c) Test  $H_0: \beta_0 = 0$  vs  $H_1: \beta_0 \neq 0$ .

(d) Compute the sum of square errors.