King Saud University	College of Sciences	Department of Mathematics	
Second Examination	Math 228	Semester 2 (1443)	Time:1H30

Exercise 1: (11 marks)

1. Let $f(x, y) = \sin(2x - 3y)$.
   (a) Evaluate $f(\frac{\pi}{4}, \frac{\pi}{3})$. (1 mark)
   (b) Find the domain of f. (1 mark)
   (c) Find the $f_x(x, y)$ and $f_y(x, y)$. (3 marks)
2. Let $g(x, y) = x^2 + y^2 + x^2y + 4$.
   (a) Find the $g_x(x, y)$ and $g_y(x, y)$. (3 marks)
   (b) Find the local maximum and minimum values and saddle points of g. (3 marks)

Exercise 2: (14 marks)

1. Calculate the following integral (2 marks)

$$\int_0^{\frac{\pi}{6}} \int_0^{\frac{\pi}{2}} (\sin x + \sin y)\, dy\, dx$$

2. Find the volume of the solid that lies under the hyperbolic paraboloid $z = 3y^2 - x^2 + 2$ and above the rectangle $R = [-1, 1] \times [1, 2]$. (3 marks)
3. Evaluate the following integral (3 marks)

$$\int_0^1 \int_0^{x^2} \cos(x^3)\, dy\, dx$$

4. Evaluate the integral $\iint_R (2x - y)\, dA$, by changing to polar coordinates, where R is the region in the first quadrant enclosed by the circle $x^2 + y^2 = 4$ and the lines $x = 0$ and $y = x$. (3 marks)
5. Use polar coordinates to find the volume of the solid that lies under the paraboloid $z = x^2 + y^2$, above the xy-plane, and inside the circle $x^2 + y^2 = 2y$. (3 marks)

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Exercice 1:

1)

a) $f\left(\frac{\pi}{4}, \frac{\pi}{3}\right) = \sin\left(2\frac{\pi}{4} - 3\frac{\pi}{3}\right) = \sin\left(-\frac{\pi}{2}\right) = -1.$ 1

b) domain of f is $\mathbb{R}^2$. 1

c) $f_x(x,y) = 2\cos(2x-3y)$

$f_y(x,y) = -3\cos(2x-3y)$. 3

2)

a) $g_x(x,y) = 2x + 2xy$. 3

$g_y(x,y) = 2y + x^2$

b) $\begin{cases} g_x(x,y) = 0 \\ g_y(x,y) = 0 \end{cases} \Rightarrow \begin{cases} 2x + 2xy = 0 \\ 2y + x^2 = 0 \end{cases} \Rightarrow \begin{cases} x(1+y) = 0 \\ 2y + x^2 = 0 \end{cases}$

(1) $\Rightarrow x = 0$ or $y = -1$.

(2) $x = 0 \Rightarrow y = 0$.

$y = -1 \Rightarrow x^2 = 2 \Rightarrow x = \sqrt{2}$ or $x = -\sqrt{2}$.

$\Rightarrow$ the critical points are $(0,0)$, $(\sqrt{2}, -1)$, $(-\sqrt{2}, 1)$ 3

$g_{xx}(x,y) = 2 + 2y$; $g_{yy}(x,y) = 2$;

$g_{xy}(x,y) = 2x$.

$\Rightarrow D(x,y) = (2+2y) \times 2 - 4x^2 = 4 + 4y - 4x^2$.

- $(0,0)$: $D(0,0) = 4 > 0$.
  and $g_{xx}(0,0) = 2 > 0 \Rightarrow g(0,0)$ is a local minimum.
- $(\sqrt{2}, -1)$: $D(\sqrt{2}, -1) = 4 - 4 - 4 \times 2 = -8 < 0$. $\Rightarrow (\sqrt{2}, -1)$ is a saddle point
- $(-\sqrt{2}, -1)$: $D(-\sqrt{2}, -1) = 4 - 4 - 4 \times 2 = -8 < 0$. $\Rightarrow (-\sqrt{2}, -1)$ " " "

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Exercise 2 :

1) $\int_0^{\frac{\pi}{6}} \int_0^{\frac{\pi}{2}} (\sin x + \sin y)\, dy\, dx$

$= \int_0^{\frac{\pi}{6}} \left[y \sin x - \cos y \right]_0^{\frac{\pi}{2}} dx$

$= \int_0^{\frac{\pi}{6}} \left[\frac{\pi}{2} \sin x + 1 \right] . dx$ 2

$= \left[-\frac{\pi}{2} \cos x + x \right]_0^{\frac{\pi}{6}}$

$= -\frac{\pi}{2} \cdot \frac{1}{2} + \frac{\pi}{6} + \frac{\pi}{2}$

$= -\frac{\pi}{4} + \frac{\pi}{6} + \frac{\pi}{2} = \frac{-3+2+6}{12}\pi = \frac{7\pi}{12}$

2).

$V = \int_{-1}^{1} \int_{1}^{2} (3y^2 - x^2 + 2)\, dy\, dx$

$= \int_{-1}^{1} \left[y^3 - yx^2 + 2y \right]_1^2 dx$ 3

$= \int_{-1}^{1} \left[8 - 2x^2 + 4 - (1 - x^2 + 2) \right] dx$

$= \int_{-1}^{1} \left[12 - 2x^2 - 3 + x^2 \right] dx$

$= \int_{-1}^{1} \left[9 - x^2 \right] dx = \left[9x - \frac{x^3}{3} \right]_{-1}^{1} = \left(9 - \frac{1}{3}\right) - \left(-9 + \frac{1}{3}\right)$

$= 18 - \frac{2}{3} = \frac{52}{3}$

3).

$\int_0^1 \int_0^{x^2} \cos(x^3)\, dy\, dx = \int_0^1 x^2 \cos(x^3)\, dx$ 3

$= \frac{1}{3} \int_0^1 3x^2 \cos x^3\, dx = \left[\frac{\sin x^3}{3} \right]_0^1 = \frac{1}{3} \sin 1 .$

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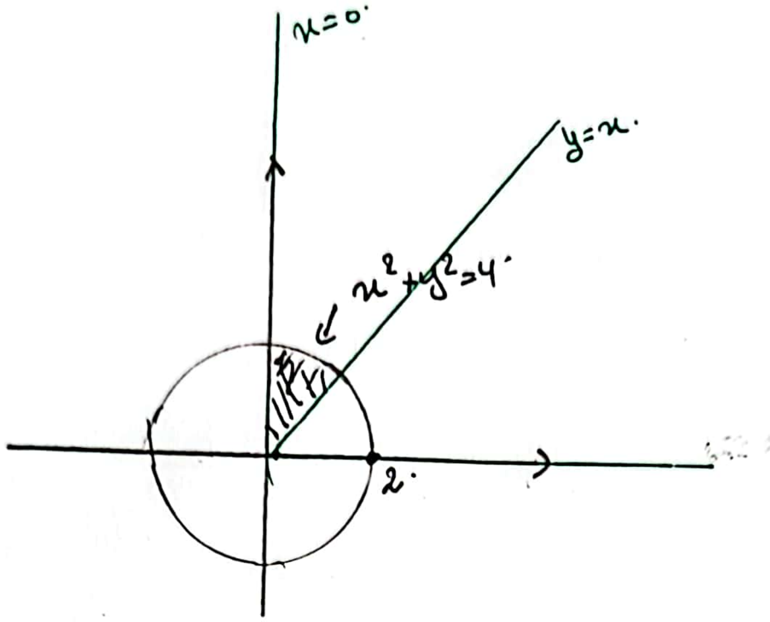
4)



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in polar coordinates the region is

$0\leq r\leq 2$ and $\frac{\pi}{4}\leq\theta\leq\frac{\pi}{2}$, $\left(\begin{array}{l} x=0 \Rightarrow r\cos\theta=0 \Rightarrow \theta=\frac{\pi}{2}. \\ x=y \Rightarrow r\cos\theta = r\sin\theta \Rightarrow \theta=\frac{\pi}{4} \end{array}\right)$.

$$\Rightarrow \iint_R (2x-y)\,dA = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\int_0^2 (2r\cos\theta - r\sin\theta)\, r\,dr\,d\theta.$$

$$= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (2\cos\theta - \sin\theta)\,d\theta \int_0^2 r^2\,dr$$

$$= \Big[2\sin\theta + \cos\theta\Big]_{\frac{\pi}{4}}^{\frac{\pi}{2}} \left[\frac{r^3}{3}\right]_0^2$$

$$= \left[(2+0) - \left(\frac{2\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\right)\right]\cdot\frac{8}{3}$$

$$= \left(2 - \frac{3\sqrt{2}}{2}\right)\cdot\frac{8}{3}.$$

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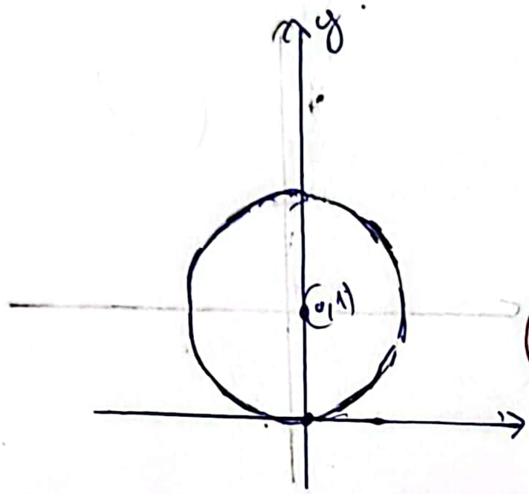
5)

$$x^2+y^2=2y \Rightarrow x^2+y^2-2y+1=1$$
$$\Rightarrow x^2+(y-1)^2=1 \cdot \quad \text{circle center } (0,1)$$
radius 1.



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$$x^2+y^2=2y \Rightarrow r^2=2r\sin\theta \Rightarrow r=2\sin\theta.$$
$$\Rightarrow 0\le r\le 2\sin\theta.$$
$$0\le\theta\le\pi.$$

$$\int_0^{\pi}\int_0^{2\sin\theta} r^2\, r\,dr\,d\theta=\int_0^{\pi}\left[\frac{r^4}{4}\right]_0^{2\sin\theta}d\theta.$$
$$=\int_0^{\pi}4\sin^4\theta\,d\theta.$$
$$=4\int_0^{\pi}\left(\frac{1-\cos 2\theta}{2}\right)^2 d\theta.$$
$$=\int_0^{\pi}(1-2\cos 2\theta+\cos^2 2\theta)\,d\theta.$$
$$=\int_0^{\pi}d\theta-2\int_0^{\pi}\cos 2\theta\,d\theta+\int_0^{\pi}\frac{1+\cos 4\theta}{2}\,d\theta.$$
$$=\pi-2\cdot\frac{1}{2}\left[\sin 2\theta\right]_0^{\pi}+\frac{1}{2}\left[\theta+\frac{1}{4}\sin 4\theta\right]_0^{\pi}.$$
$$=\pi-0+\frac{1}{2}\pi=\frac{3\pi}{2}.$$

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