

MIDTERM, SEMESTER I, 1445
DEPT. OF MATHEMATICS, COLLEGE OF SCIENCE, KSU
MATH: 280 FULL MARK: 25 TIME: 1H:30

Question 1[4+4]

- 1) Show that $\sup\{\frac{2n^2}{3n^2+1} : n \in \mathbb{N}\} = \frac{2}{3}$.
- 2) Find $\sup(-\sqrt{2}, \sqrt{2}) \cap \mathbb{Q}$.

Question 2 [5] Define the sequence (x_n) by

$$x_1 = 1, \quad x_{n+1} = \sqrt{1 + x_n}.$$

Show that (x_n) is convergent and determine its limit.

Question 3[4+4+4]

- 1) Show that if a series $\sum_{n=1}^{\infty} a_n$ is convergent, then $\lim_{n \rightarrow \infty} a_n = 0$.
- 2) Give an example of a convergent series which is not absolutely convergent.
- 3) Test the following series for convergence:

$$i) \sum_{k=1}^{\infty} \frac{\sin k}{(1.001)^k}, \quad ii) \sum_{k=1}^{\infty} \frac{k^2 + 3}{4k^2 + 5}, \quad iii) \sum_{k=1}^{\infty} \frac{e^{-k}}{k^k}.$$