Kingdom of Saudi Arabia
Ministry of Education
King Saud University
College of Science
Department of mathematics

المملكة العربية السعودية
وزارة التعليم
جامعة الملك سعود
كلية العلوم
قسم الرياضيات

MIDTERM 1 EXAM

SEMESTER	SECOND TERM	YEAR	2017/2016
	COURSE	ACTU 466	
DATE	15/03/2017	DURATION	1H 30 MNS

	رقم الشعبة:		إســم الطالب(ة):
	توقيع الطالب(ة):		الرقم الجامعي للطالب(ة):

INSTRUCTIONS

1) Please check that your exam contains **07 pages** total (including the first page!!), **08 questions**.
2) **Answer all questions.**
3) No books, No notes and no phones are allowed.
4) A standard no programmable calculator is allowed.
5) Table for most used distributions is included.
6) Z-table is included.

Question	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	
Total score	**2**	**2**	**4**	**4**	**3**	**2**	**2**	**6**	
Score									

1) (2 marks) Consider the density function:

$$f(x) = \begin{cases} 4x^3 & if \quad 0 < x < 1 \\ 0 & if\ not \end{cases}$$

Compute the median and the 0.23 quantile.

2) (2 marks) Consider the following mass function:

x	0	1	2	3	4	5	6
$f(x)$	0.1	0.15	0.15	0.2	0.2	0.1	0.1

Compute the median and the 80th percentile.

3) (4 marks) Let $X \sim Exponential(\delta)$ and define $Y = e^X$.

a) Compute the cdf and the pdf of Y.

b) Determine whether the distribution of Y is light-tailed or heavy-tailed, using

- the method of moments.
- the hazard rate function method.

4) (4 marks) Let $X \sim N(0,1)$ with density function ϕ.

a) Show that the tail-value-at-risk of X at $100p\%$, is given by:

$$TVaR_p(X) = \frac{1}{1-p}\phi(VaR_p(X)).$$

b) Compute $VaR_p(X)$ and $TVaR_p(X)$ for $p = 0.95$.

5) (3 marks) Let X have a cdf $F(x) = 1 - (1 + x)^{-\alpha}$ where $x > 0$ and $\alpha > 0$. Determine the pdf and the cdf of $Y = cX$ with $c > 0$.

6) (2 marks) Let $X \sim Uniform(0, a)$. Find the pdf of $Y = X^{\frac{1}{\tau}}$ with $\tau > 0$.

7) (2 marks) Show that the binomial distribution with parameters m and p, belongs to the linear exponential family with respect to the parameter $\theta = p$.

8) (6 marks) Let $N \sim Poisson(\theta)$ and let N^T be the zero-truncated random variable associated to N. Compute:

a) $p^T(k)$ for $k = 0$ and $k \geq 1$.
b) the mean of N^T.
c) the mgf of N^T.

Table A The most frequently used discrete and continuous distributions

Distribution	Density & support	Moments & cumulants	Mgf
Binomial(n,p) $(0<p<1,\ n\in\mathbb{N})$	$\binom{n}{x}p^x(1-p)^{n-x}$ $x=0,1,\ldots,n$	$\mathrm{E}=np, \mathrm{Var}=np(1-p)$, $\gamma=\frac{np(1-p)(1-2p)}{\sigma^3}$	$(1-p+pe^t)^n$
Bernoulli(p)	$\equiv$ Binomial$(1,p)$		
Poisson(λ) $(\lambda>0)$	$e^{-\lambda}\frac{\lambda^x}{x!}, x=0,1,\ldots$	$\mathrm{E}=\mathrm{Var}=\lambda$, $\gamma=1/\sqrt{\lambda}$, $\kappa_j=\lambda, j=1,2,\ldots$	$\exp[\lambda(e^t-1)]$
Negative binomial(r,p) $(r>0,\ 0<p<1)$	$\binom{r+x-1}{x}p^r(1-p)^x$ $x=0,1,2,\ldots$	$\mathrm{E}=r(1-p)/p$ $\mathrm{Var}=\mathrm{E}/p$, $\gamma=\frac{(2-p)}{p\sigma}$	$\left(\frac{p}{1-(1-p)e^t}\right)^r$
Geometric(p)	$\equiv$ Negative binomial$(1,p)$		
Uniform(a,b) $(a<b)$	$\frac{1}{b-a}; a<x<b$	$\mathrm{E}=(a+b)/2$, $\mathrm{Var}=(b-a)^2/12$, $\gamma=0$	$\frac{e^{bt}-e^{at}}{(b-a)t}$
$\mathrm{N}(\mu,\sigma^2)$ $(\sigma>0)$	$\frac{1}{\sigma\sqrt{2\pi}}\exp\frac{-(x-\mu)^2}{2\sigma^2}$	$\mathrm{E}=\mu$, $\mathrm{Var}=\sigma^2$, $\gamma=0$ $(\kappa_j=0,\ j\geq 3)$	$\exp(\mu t+\frac{1}{2}\sigma^2t^2)$
Gamma(α,β) $(\alpha,\beta>0)$	$\frac{\beta^\alpha}{\Gamma(\alpha)}x^{\alpha-1}e^{-\beta x}, x>0$	$\mathrm{E}=\alpha/\beta$, $\mathrm{Var}=\alpha/\beta^2$, $\gamma=2/\sqrt{\alpha}$	$\left(\frac{\beta}{\beta-t}\right)^\alpha$ $(t<\beta)$
Exponential(β)	$\equiv$ gamma$(1,\beta)$		
$\chi^2(k)$ $(k\in\mathbb{N})$	$\equiv$ gamma$(k/2,1/2)$		
Inverse Gaussian(α,β) $(\alpha>0,\beta>0)$	$\frac{\alpha x^{-3/2}}{\sqrt{2\pi\beta}}\exp\left(\frac{-(\alpha-\beta x)^2}{2\beta x}\right)$	$\mathrm{E}=\alpha/\beta$, $\mathrm{Var}=\alpha/\beta^2$, $\gamma=3/\sqrt{\alpha}$	$e^{\alpha(1-\sqrt{1-2t/\beta})}$ $(t\leq\beta/2)$

$$F(x)=\Phi\left(\frac{-\alpha}{\sqrt{\beta x}}+\sqrt{\beta x}\right)+e^{2\alpha}\Phi\left(\frac{-\alpha}{\sqrt{\beta x}}-\sqrt{\beta x}\right),\quad x>0$$

Standard Normal Probabilities

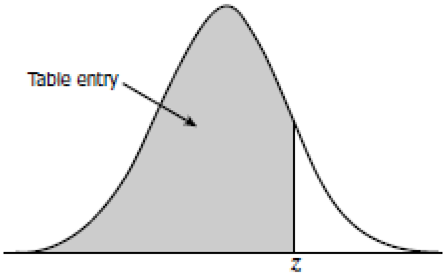

Table entry for z is the area under the standard normal curve to the left of z.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998