106Midterm1 solutions(Sem1-37/38)

Question1 a)

$F'(x) = coshx.\int_0^{2x}(1+t^2)^5dt + 2sinhx(1+4x^2)^5 \quad (1) F'(0) = 0$

b) $\int_0^{\pi/4}\frac{(1+tanx)^6}{cos^2x}dx = \int_0^{\pi/4}(1+tanx)^6 sec^2xdx$

$= \int_1^2 u^6du \qquad u = 1+tanx, du = sec^2xdx$

$= [u^7/7]_1^2 = \frac{1}{7}(2^7-1)$

c) $\int_{-1}^2\sqrt{x+2}\,dx = \frac{2}{3}[(x+2)^{\frac{3}{2}}]_{-1}^2 = \frac{14}{3}$

$\frac{14}{9} = \sqrt{c+2}$. So $c = \frac{34}{81}$

Question2 a)$x_0 = 0, x_1 = \frac{1}{4}, x_2 = \frac{1}{2}, x_3 = \frac{3}{4}, x_4 = 1$

$T_4 = \frac{1}{8}\left(1 + 2\sqrt{\frac{17}{16}} + 2\sqrt{\frac{5}{4}} + 2\frac{5}{4} + \sqrt{2}\right)$

$T_4 \approx \frac{1}{8}(1 + 2.061553 + 2.236068 + 2.5 + 1.414213) \approx 1.151479$

b) $f'(x) = \frac{(cosh^{-1}(2x))'}{ln10.cosh^{-1}(2x)} = \frac{2}{ln10.cosh^{-1}(2x).\sqrt{4x^2-1}}$

Question3 a) $y = \frac{x^x.\sqrt[3]{1+4x}}{sin^{-1}x}$

$lny = xlnx + \frac{1}{3}ln(1+4x) - ln\left(sin^{-1}(x)\right)$

$\frac{y'}{y} = lnx + 1 + \frac{4}{3(1+4x)} - \frac{1}{sin^{-1}(x).\sqrt{1-x^2}}$

$$y' = \left(lnx + 1 + \frac{4}{3(1+4x)} - \frac{1}{sin^{-1}(x).\sqrt{1-x^2}} \right) y$$

b) $\int \frac{2xln\,(1+x^2)dx}{1+x^2} = \int udu \qquad u = \ln(1+x^2) \quad du = \frac{2xdx}{1+x^2}$

$$= \frac{u^2}{2} + C = \frac{1}{2}(\ln(1+x^2))^2 + C$$

<u>Question4</u>

a) $\int \frac{3^x dx}{2+3^{2x}} = \frac{1}{ln3} \int \frac{du}{2+u^2} \qquad u = 3^x\,,\; du = ln3.\,3^x dx$

$$= \frac{1}{\sqrt{2}ln3} tan^{-1}\left(\frac{u}{\sqrt{2}}\right) + C = \frac{1}{\sqrt{2}ln3} tan^{-1}\left(\frac{3^x}{\sqrt{2}}\right) + C$$

b) $\int \frac{dx}{\sqrt{9-e^{6x}}} = \frac{1}{3} \int \frac{du}{u\sqrt{3^2-u^2}} \qquad u = e^{3x} \;\; dx = \frac{du}{3u}$

$$= -\frac{1}{9} sech^{-1}\left(\frac{e^{3x}}{3}\right) + C$$