Grading scheme for midterm1

Q1) **a)** $F'(x) = 2(4-4x^2)^{1/3} - cosx.(4-(sinx)^2)^{1/3}$ **(1) + (1)**

b) $\int \frac{\sqrt{\sqrt{x}-4}}{\sqrt{x}}dx = 2\int \sqrt{u}du \quad u = \sqrt{x}-4$ **(2)**

$= \frac{4}{3}u^{3/2} + C = \frac{4}{3}(\sqrt{x}-4)^{3/2} + C$ **(1)**

c) $\sum_1^9 (3k^2 - k + M) = 3\sum_1^9 k^2 - \sum_1^9 k + 9M = 909M$ **(1)**

$3\frac{9.10.19}{6} - \frac{9.10}{2} = 900M$ **(1)**

$M = \frac{9}{10}$ **(1)**

Q2 **a)** $P = \left\{0, \frac{3}{n}, \frac{6}{n}, \ldots\ldots, 3\right\}$

$x_k = \frac{3k}{n}, 0 \le k \le n, \Delta x_k = \frac{3}{n}, 1 \le k \le n$ **(1)**

Take $u_k = x_k = \frac{3k}{n} \quad 1 \le k \le n$ **(0.5)**

$R_P = \sum_{k=1}^{n}(9 + 9\frac{k^2}{n^2})\frac{3}{n} = 27 + 27\frac{n(n+1)(2n+1)}{6n^3} \to 36 \text{ as } n \to \infty$ **(1.5)**

So $\int_0^3 (9+x^2)dx = 36$

b) $P = \{0,1,2,3,4,5,6\} \quad S_6 = \frac{6}{3\times 6}\left(\frac{1}{2} + \frac{4}{3} + \frac{2}{4} + \frac{4}{5} + \frac{2}{6} + \frac{4}{7} + \frac{1}{8}\right)$ **(0,5) + (2)**

$S_6 \approx \frac{1}{3} \times 4.16309 \approx 1.38769$ **(0,5)**

c) $y' = \pi x^{\pi-1} + ln\pi . \pi^x + (lnx+1). x^x \quad (\mathbf{0,5}) + (\mathbf{0,5}) + (\mathbf{1})$

Q3) a) $\int \frac{3x+(\tan^{-1} x)^2}{1+x^2} dx = \frac{3}{2}\int \frac{2x}{1+x^2} dx + \int \frac{(\tan^{-1} x)^2}{1+x^2} dx \quad (\mathbf{1})$

$$= \frac{3}{2} ln(1+x^2) + \frac{1}{3}(tan^{-1} x)^3 + C \quad (\mathbf{2})$$

b) $\int \frac{\sqrt{x}dx}{\sqrt{1+x^3}} = \int \frac{\sqrt{x}}{\sqrt{1+(x^{3/2})^2}} dx = \frac{2}{3}\int \frac{du}{\sqrt{1+u^2}} \quad u = x^{3/2} \quad (\mathbf{2})$

$$= \frac{2}{3} sinh^{-1}(x^{3/2}) + C \quad (\mathbf{1})$$

c) $\int \frac{dx}{\sqrt{3^{2x}-9}} = \int \frac{dx}{\sqrt{(3^x)^2-3^2}} = \frac{1}{ln3}\int \frac{du}{u\sqrt{u^2-3^2}} \quad u = 3^x \quad (\mathbf{2})$

$$= \frac{1}{3ln3} \sec^{-1}(3^{x-1}) + C \quad (\mathbf{1})$$