Final Exam, Semester II, 1445
Dept. of Mathematics, College of Science, KSU
Math: 280 — Full Mark: 40 — Time: 3H

immediate

Question 1 [2+2+2]

1. Prove that for every real number, there exists an integer n such that $n-1 \leq x < n$. Find such n if $x = -\frac{17}{5}$.
2. Determine $\sup(A)$ and $\inf(A)$ where $A = \{x \in \mathbb{R} : x^2 - 9 < 0\}$, and justify your answer.
3. Show that $\sup\{\frac{n^2}{n^2+1} : n \in \mathbb{N}\} = 1$.

Question 2 [4+4]

Find the following limits, if they exist:
1. $\lim_{n\to\infty} \frac{n^3}{2n^4+1}$.
2. $\lim_{n\to\infty} c^{\frac{1}{n}}$, where $c > 1$.
3. $\lim_{n\to\infty} n^{\frac{1}{n}}$, where $n \in \mathbb{N}$.
4. $\lim_{n\to\infty} na^n = 0$, where $0 < a < 1$.

Question 3

Discuss the convergence of the following series:
(i) $\sum_{n=1}^{\infty} \frac{(-1)^n \sqrt{n}}{n^2+1}$
(ii) $\sum_{n=1}^{\infty} \frac{2^n n!}{n^n}$

Question 4

Find the following limits, if they exist, and prove using the definition of the limit or sequence characterization:
(i) $\lim_{x\to 0} \frac{x^2}{|x|}$

(ii) $\lim_{x\to\infty}(\text{sign}(x) + x)$
(iii) $\lim_{x\to-\infty} \frac{x}{\sqrt{x^2+2}}$
(iv) $\lim_{x\to\infty} \frac{x^4}{e^x}$

Question 5

1. State Rolle's theorem
2. Prove that if f is continuous on $[a, b]$ and has zero derivative on (a, b), then f is constant.
3. Approximate the function $f(x) = \sin x$ on $(-1, 1)$ by a polynomial of degree 3.

Question 6

Let f be a bounded function on $[a, b]$.
(i) Prove that if f is integrable, then f.
(ii) Is the converse of (i) true? Justify your answer.
[Your justification goes here.]
(iii) Evaluate $\int_0^1 x^3\,dx$ using Riemann sums.
[Your solution goes here.]