Kingdom of Saudi Arabia
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المملكة العربية السعودية
وزارة التعليم
جامعة الملك سعود
كلية العلوم
قسم الرياضيات

MIDTERM 2 EXAM

SEMESTER	SECOND TERM	YEAR	2017/2016
	COURSE	ACTU 465	
DATE	26/04/2017	DURATION	1H 30 MNS

	رقم الشعبة:		إســم الطالب(ة):
	توقيع الطالب(ة):		الرقم الجامعي للطالب(ة):

INSTRUCTIONS

1) Please check that your exam contains **07 pages** total (including the first page!!), **04 questions and a Bonus question**.
2) **Answer all questions.**
3) No books, No notes and no phones are allowed.
4) A standard no programmable calculator is allowed.
5) Table for most used distributions is included.

Question	**1**	**2**	**3**	**4**	**BQ**	
Total score	**6**	**6**	**7**	**6**	**3**	
Score						

Exercise 1. (2+2+2=6 marks) Let $X_1, \ldots, X_n$ be past claim amounts. Suppose that $X_i|\Theta$ are independent and identically uniformly distributed on the interval $(0, \Theta)$ and Θ is Gamma distributed with parameters α and β.

Determine

a) the hypothetical mean, its mean and variance.
b) the process variance and its mean.
c) the Buhlmann premium.

Exercise 2. (2+2+2=6 marks) Suppose the conditional distribution of the number of claims and the prior distribution are given as follows:

$X\|\Theta$	Probability
0	$\Theta/10$
1	$\Theta/5$
2	$1-3\Theta/10$

Θ	Probability
1	0.3
2	0.7

Suppose further that a randomly chosen insured has one claim in year 1 and 2 claims in year 2. Determine

a) the hypothetical mean, its mean and variance.
b) the process variance and its mean.
c) the Buhlmann estimate for the number of claims in year 3.

Exercise 3. (2+2+3=7 marks)

You are given:

(i) The number of claims incurred in a year by any insured has a Binomial distribution with parameters m and q.

(ii) The claim frequencies of different insureds are independent.

(iii) The prior distribution M is Geometric with parameter p.

(iv)

Year	Annual Number of insureds	Annual Number of claims
1	120	10
2	100	8
3	180	14
4	200	?

1) Determine
a) the hypothetical mean, its mean and variance.
b) the process variance and its mean.
2) Suppose $p = q = 0.2$, determine the Buhlmann-Straub credibility estimate of the number of claims in Year 4.

Exercise 4. (2+2+2=6 marks)

You are given total claims for two policyholders:

Policyholder	Year 1	Year 2	Year 3	Year 4
X	730	800	650	700
Y	655	650	625	750

Using the nonparametric empirical Bayes method, determine the estimated value of

a) the mean and variance of the hypothetical mean.
b) the mean of the process variance.
c) the Buhlmann credibility premium for Policyholder Y.

Bonus Question. (3 marks)
You are given $X_1, \dots, X_n$ such that:
(i) The model distribution of $X_i|M$ is Poisson with parameter M.
(ii) The prior distribution of M is exponential with parameter δ.
Show that the model satisfies exact credibility.

Table A The most frequently used discrete and continuous distributions

Distribution	Density & support	Moments & cumulants	Mgf
Binomial(n,p) $(0<p<1,\ n\in\mathbb{N})$	$\binom{n}{x}p^x(1-p)^{n-x}$, $x=0,1,\dots,n$	$\mathrm{E}=np, \mathrm{Var}=np(1-p)$, $\gamma=\frac{np(1-p)(1-2p)}{\sigma^3}$	$(1-p+pe^t)^n$
Bernoulli(p)	$\equiv$ Binomial$(1,p)$		
Poisson(λ) $(\lambda>0)$	$e^{-\lambda}\frac{\lambda^x}{x!}, x=0,1,\dots$	$\mathrm{E}=\mathrm{Var}=\lambda$, $\gamma=1/\sqrt{\lambda}$, $\kappa_j=\lambda,\ j=1,2,\dots$	$\exp[\lambda(e^t-1)]$
Negative binomial(r,p) $(r>0,\ 0<p<1)$	$\binom{r+x-1}{x}p^r(1-p)^x$, $x=0,1,2,\dots$	$\mathrm{E}=r(1-p)/p$, $\mathrm{Var}=\mathrm{E}/p$, $\gamma=\frac{(2-p)}{p\sigma}$	$\left(\frac{p}{1-(1-p)e^t}\right)^r$
Geometric(p)	$\equiv$ Negative binomial$(1,p)$		
Uniform(a,b) $(a<b)$	$\frac{1}{b-a}$; $a<x<b$	$\mathrm{E}=(a+b)/2$, $\mathrm{Var}=(b-a)^2/12$, $\gamma=0$	$\frac{e^{bt}-e^{at}}{(b-a)t}$
$\mathrm{N}(\mu,\sigma^2)$ $(\sigma>0)$	$\frac{1}{\sigma\sqrt{2\pi}}\exp\frac{-(x-\mu)^2}{2\sigma^2}$	$\mathrm{E}=\mu$, $\mathrm{Var}=\sigma^2$, $\gamma=0$ $(\kappa_j=0,\ j\geq 3)$	$\exp(\mu t+\frac{1}{2}\sigma^2t^2)$
Gamma(α,β) $(\alpha,\beta>0)$	$\frac{\beta^\alpha}{\Gamma(\alpha)}x^{\alpha-1}e^{-\beta x}, x>0$	$\mathrm{E}=\alpha/\beta$, $\mathrm{Var}=\alpha/\beta^2$, $\gamma=2/\sqrt{\alpha}$	$\left(\frac{\beta}{\beta-t}\right)^\alpha$ $(t<\beta)$
Exponential(β)	$\equiv$ gamma$(1,\beta)$		
$\chi^2(k)$ $(k\in\mathbb{N})$	$\equiv$ gamma$(k/2,1/2)$		
Inverse Gaussian(α,β) $(\alpha>0,\beta>0)$	$\frac{\alpha x^{-3/2}}{\sqrt{2\pi\beta}}\exp\left(\frac{-(\alpha-\beta x)^2}{2\beta x}\right)$	$\mathrm{E}=\alpha/\beta$, $\mathrm{Var}=\alpha/\beta^2$, $\gamma=3/\sqrt{\alpha}$	$e^{\alpha(1-\sqrt{1-2t/\beta})}$ $(t\leq\beta/2)$

$$F(x)=\Phi\left(\frac{-\alpha}{\sqrt{\beta x}}+\sqrt{\beta x}\right)+e^{2\alpha}\Phi\left(\frac{-\alpha}{\sqrt{\beta x}}-\sqrt{\beta x}\right),\quad x>0$$