King Saud University

Department of Mathematics

First Mid Term Exam

ACTU 461 - Mathematics of Finance (2)

(16/5/1437 H, Time 11--12:30)

Exercice 1. [7]

1) Give two different versions of the Non-Arbitrage Principle in a market with portfolios containing stocks, bonds and forward contracts.

2) Given the bond prices $A(0)$ at $t = 0$ and $A(1)$ at $t = 1$, the stock price $S(0)$. Prove that if the risky securities involves no cost, then the forward price F must be:

$$F = S(0)(1 + K_A) \text{ where } K_A \text{ is the return on bond,}$$

or an arbitrage opportunity would exist otherwise.

3) Let $A(0) = 100, A(1) = 112$ and $S(0) = 34$ SAR. Is it possible to find an arbitrage opportunity if the forward price of stock is $F = 38.60$ SAR

Exercise2. [5]

Let consider the following information about bond and stock prices:

A(0)= 80; A(1)=100, S(0)=80 dollars and

$$S(1) = \begin{cases} 110, & \text{with probability } p \\ 60, & \text{with probability } 1 - p \end{cases}$$

Compute the price $C(0)$ of a call option with strike price $ 90.

Exercise3. [5]

We consider:

- Annuity A pays 1 at the beginning of each year for three years.
- Perpetuity B pays 2 at the beginning of each year.

We suppose that the Macaulay duration of Annuity A at the time of purchase is 0.93 and Annuity A offer the same yield as Perpetuity B.

Calculate the Macaulay duration of Perpetuity B at the time of purchase.

Question 4. [8]

1) You are giving the following information about a bond A:
   - The term to maturity is 2 years.
   - The coupon rate is 6% paid semi-annually
   - The yield rate is 8% convertible quarterly
   - The par value is 100 SAR

a) Calculate the Macaulay duration of the bond A.
b) Calculate the convexity of the bond A.
c) We suppose that the yield rate increases to 8.4%. Determine the amount of error in using duration and convexity to estimate the price.

2) We consider a 3 years- bond B with 5 % annual coupon rate, 4,75 % annual yield rate and par value 100 SAR.Compare which bond A or B is more sensitive to interest rate changes.