Question 1. a) [7] Show that $f(x) = \cosh 2x$, $g(x) = e^{-2x}$ are solutions of the differential equation: $y'' - 4y = 0$, and show that $\{\cosh 2x, e^{-2x}\}$ is a fundamental set of solutions of the differential equation: $y'' - 4y = 0$.

b) [8] If the function $y_1 = x$ is a solution of the differential equation

$$x^2y'' - (x^2 + 2x)y' + (x + 2)y = 0, \quad x > 0.$$ 

Use the method of reduction of order to find the second solution $y_2$, and hence find the general solution.

Question 2. [8] Use the method of variation of parameters to find the general solution of the nonhomogeneous differential equation:

$$y'' - 2y' + 2y = e^x \tan x.$$ 

(Hint: $\int \sec ax dx = \frac{1}{a} \ln |\sec ax + \tan ax|.$)

Question 3. [8] Determine only the form of the particular solution of the differential equation

$$y''' - 2y'' + y' = 7e^{-x} \sin 2x.$$ 

Question 4. [9] Solve the system of differential equations

$$\begin{align*}
\frac{dx}{dt} + 2y &= -x \\
2\frac{dz}{dt} + 2w &= -y.
\end{align*}$$