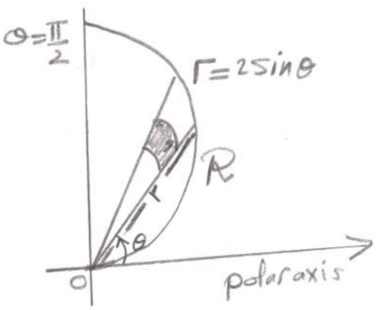
Q1 Use polar coordinates to evaluate the double integral

$$\int_0^2 \int_0^{\sqrt{2y-y^2}} \sqrt{4-x^2-y^2}\;dx\,dy$$

Ans:

$$\int_0^2 \int_0^{\sqrt{2y-y^2}} \sqrt{4-x^2-y^2}\;dx\,dy$$

$$= \int_0^{\pi/2} \int_0^{2\sin\theta} \sqrt{4-r^2}\; r\,dr\,d\theta$$



$x = \sqrt{2y - y^2}$
$x^2 = 2y - y^2$
$x^2 + y^2 = 2y$
$r^2 = 2r\sin\theta$
$r = 2\sin\theta$ circle
$0 < r < 2\sin\theta$

$$= \frac{-1}{2} \int_0^{\pi/2} \int_0^{2\sin\theta} (4-r^2)^{\frac{1}{2}} \cdot (-2r)\,dr\,d\theta$$

$$= \frac{-1}{2} \int_0^{\pi/2} \left[\frac{(4-r^2)^{3/2}}{3/2} \right]_0^{2\sin\theta} d\theta$$

$$= \frac{-1}{3} \int_0^{\pi/2} (8\cos^3\theta - 8)\,d\theta$$

$$= \frac{8}{3} \int_0^{\pi/2} (1 - \cos^3\theta)\,d\theta$$

$$= \frac{8}{3} \left[\frac{\pi}{2} - \int_0^{\pi/2} \cos^3\theta\,d\theta \right] \quad (1)$$

$$\int_0^{\pi/2} \cos^3\theta\,d\theta = \int_0^{\pi/2} \cos^2\theta \cdot \cos\theta\,d\theta$$

$$= \int_0^{\pi/2} (1 - \sin^2\theta) \cdot \cos\theta\,d\theta$$

P.2

$$\int_0^{\pi/2} \cos^3\theta \, d\theta = \left[\sin\theta - \frac{\sin^3\theta}{3}\right]_0^{\pi/2}$$

$$= 1 - \frac{1}{3} = \frac{2}{3} \qquad (2)$$

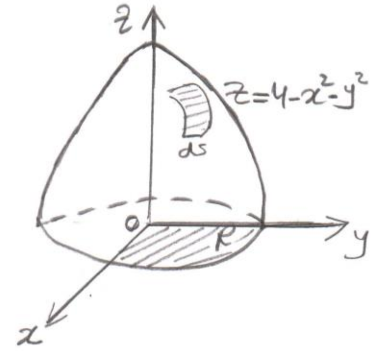
Subs. (2) in (1)

$$\Rightarrow \int_0^2 \int_0^{\sqrt{2y-y^2}} \sqrt{4-x^2-y^2}\, dx\, dy$$

$$= \frac{8}{3}\left[\frac{\pi}{2} - \frac{2}{3}\right] = \frac{4}{9}(3\pi - 4)$$

$$\simeq 2.4$$

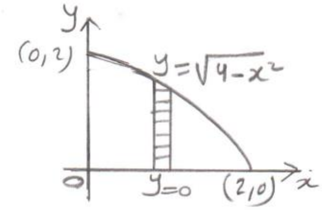
Q 2 Find the surface area of the paraboloid given by $z = 4 - x^2 - y^2$, $z \geq 0$



Ans:

$$S.A = \iint_R \sqrt{1 + f_x^2 + f_y^2}\, dA$$

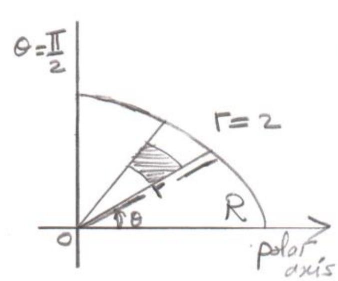
$$S.A = 4 \int_0^2 \int_0^{\sqrt{4-x^2}} \sqrt{1 + 4x^2 + 4y^2}\, dy\, dx$$



$$= 4 \int_0^{\pi/2} \int_0^2 \sqrt{1 + 4r^2}\, r\, dr\, d\theta$$

$$= \frac{4}{8} \int_0^{\pi/2} \int_0^2 (1 + 4r^2)^{\frac{1}{2}} \cdot 8r\, dr\, d\theta$$

$$= \frac{1}{2} \int_0^{\pi/2} \left[\frac{(1+4r^2)^{3/2}}{3/2} \right]_0^2 d\theta$$

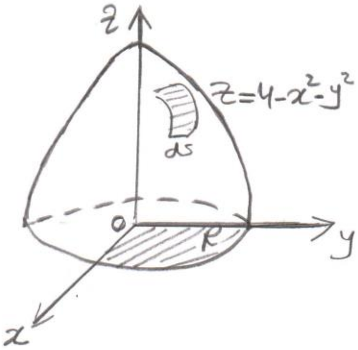


$$= \frac{1}{3} \left[(17)^{3/2} - 1 \right] \int_0^{\pi/2} d\theta$$

$$\therefore\ S.A = \frac{\pi}{6} \left[(17)^{3/2} - 1 \right] \simeq 36.177$$

P.3

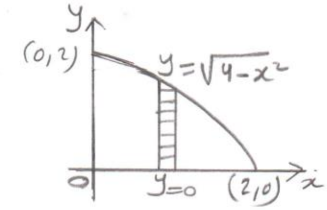
Q 2 Find the surface area of the paraboloid given by $z = 4 - x^2 - y^2, \quad z \geq 0$



Ans:

$$S.A = \iint_R \sqrt{1 + f_x^2 + f_y^2}\, dA$$

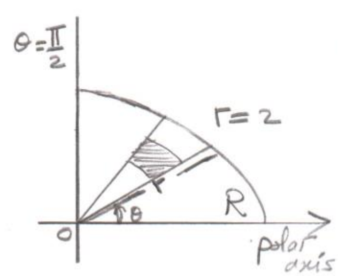
$$S.A = 4 \int_0^2 \int_0^{\sqrt{4-x^2}} \sqrt{1 + 4x^2 + 4y^2}\, dy\, dx$$



$$= 4 \int_0^{\pi/2} \int_0^2 \sqrt{1 + 4r^2}\, r\, dr\, d\theta$$

$$= \frac{4}{8} \int_0^{\pi/2} \int_0^2 (1 + 4r^2)^{\frac{1}{2}} \cdot 8r\, dr\, d\theta$$

$$= \frac{1}{2} \int_0^{\pi/2} \left[\frac{(1+4r^2)^{3/2}}{3/2} \right]_0^2 d\theta$$



$$= \frac{1}{3} \left[(17)^{3/2} - 1 \right] \int_0^{\pi/2} d\theta$$

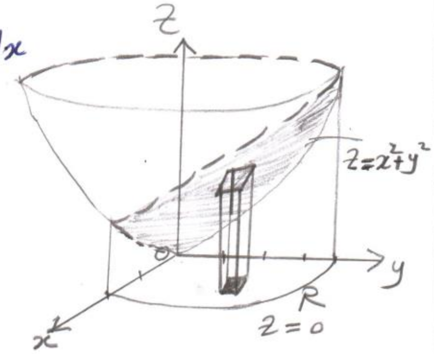
$$\therefore \quad S.A = \frac{\pi}{6} \left[(17)^{3/2} - 1 \right] \simeq 36.177$$

Q3: P.4 Find the volume of the solid in the first octant bounded by the graphs of

$z = x^2 + y^2$ and $y = 4 - x^2$

Ans:

$$V = \iiint_Q dV = \int_0^2 \int_0^{4-x^2} \int_0^{x^2+y^2} dz\,dy\,dx$$

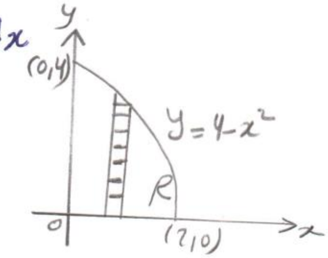


$$= \int_0^2 \int_0^{4-x^2} (x^2+y^2)\,dy\,dx$$

$$= \int_0^2 \left[x^2 y + \frac{y^3}{3} \right]_0^{4-x^2} dx$$

$$= \int_0^2 \left[x^2(4-x^2) + \frac{(4-x^2)^3}{3} \right] dx$$

$$= \int_0^2 \left[4x^2 - x^4 + \frac{1}{3}(64 - 48x^2 + 12x^4 - x^6) \right] dx$$



$$= \int_0^2 \left(-12x^2 + 3x^4 + \frac{64}{3} - \frac{1}{3}x^6 \right) dx$$

$$= \left[-4x^3 + \frac{3x^5}{5} + \frac{64}{3}x - \frac{x^7}{21} \right]_0^2$$

$$V = \frac{832}{35} \simeq 23.77$$

- Another Solution

$$V = \iint_R (x^2+y^2)\,dA = \int_0^2 \int_0^{4-x^2} (x^2+y^2)\,dy\,dx$$

- - - - -

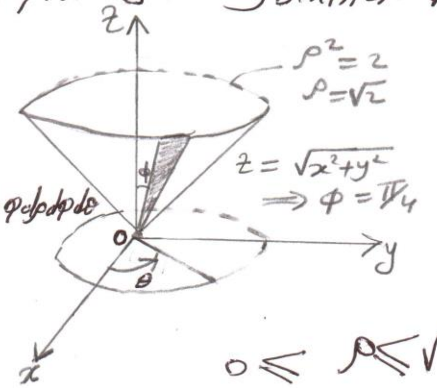
P.5

Q4 Use spherical Coords to find the mass of the solid that lies above the Cone $z = \sqrt{x^2+y^2}$ and below $x^2+y^2+z^2=2$, having density $\delta(x,y,z) = \sqrt{x^2+y^2+z^2}$

sphere

Ans:

mass, $m = \int_0^{2\pi}\int_0^{\pi/4}\int_0^{\sqrt{2}} \rho \cdot \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$



$$m = \int_0^{2\pi}\int_0^{\pi/4}\int_0^{\sqrt{2}} \rho^3 \sin\phi \, d\rho \, d\phi \, d\theta$$

$0 \leq \rho \leq \sqrt{2}$

$0 \leq \theta \leq 2\pi$

$0 \leq \phi \leq \frac{\pi}{4}$

$$m = \int_0^{2\pi}\int_0^{\pi/4} \underbrace{\left[\frac{\rho^4}{4}\right]_0^{\sqrt{2}}}_{1} \sin\phi \, d\phi \, d\theta$$

$$m = \int_0^{2\pi}\int_0^{\pi/4} \sin\phi \, d\phi \, d\theta$$

$$m = \int_0^{2\pi} \left[-\cos\phi\right]_0^{\pi/4} d\theta$$

$$m = \left(\frac{-1}{\sqrt{2}} + 1\right) \int_0^{2\pi} d\theta$$

$$m = 2\pi\left(1 - \frac{1}{\sqrt{2}}\right) = (2-\sqrt{2})\pi$$

$$m \simeq 1.84$$