P1

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Model answer of Mid II

55/1438

Q1 a) $a_2(x) = 4x^2 - 4$ is Continuous $\forall x \in \mathbb{R}$

$a_1(x) = \sqrt{4x^2-4}$ " " $\forall x \in \mathbb{R}$ and $|x| \geq 1$

$\therefore$ $a_2(x)$ and $a_1(x)$ are Continuous on $(-\infty, -1] \cup [1, \infty)$

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$a_2(x) = 0$ if $x = \pm 1$

$a_2(x) \neq 0$ $\forall x \in (1, \infty)$ or $(-\infty, -1)$

but $\because$ $x_0 = 4 \in (1, \infty)$

$\therefore$ the Largest interval for which the IVP has a unique Solution is $(1, \infty)$

b) $x^2 y'' - x y' + 2y = 0$

$\Rightarrow y'' - \frac{1}{x} y' + \frac{2}{x^2} y = 0$, $x \neq 0$

$\Rightarrow p(x) = \frac{1}{x}$

$y_2(x) = x \sin(\ln x) \int \frac{e^{-\int \frac{-1}{x} dx}}{x^2 \sin^2(\ln x)} dx$

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$y_2 = x \sin(\ln x) \int \frac{e^{\ln x}}{x^2 \sin^2(\ln x)} dx$

$y_2 = x \sin(\ln x) \int \csc^2(\ln x) \cdot \frac{1}{x} dx$

$y_2 = x \sin(\ln x) [-\cot(\ln x)]$

$y_2 = -x \cos(\ln x)$

$\therefore$ the general Solution of the given DE is

$y = c_1 x \sin(\ln x) + c_2 x \cos(\ln x)$ #

P 2

Q2 (8) a) $y''' + 2y'' + y' = 5 - 4\cos x + 3e^{x} + 5e^{-x}$

Ans: we have $y_c = c_1 + c_2 e^{-x} + c_3 x e^{-x}$

(4) and the particular soln has the form

$y_p = Ax + B\cos x + C\sin x + De^{x} + Ex^2 e^{-x}$

b) $x^2y'' + 4xy' + 2y = \ln x$, $x > 0$ Cauchy Euler Eqn

For the homogeneous DE $x^2y'' + 4xy' + 2y = 0$

we have $y_c = c_1 x^{-1} + c_2 x^{-2}$

To find the particular soln, we have

$$W = \begin{vmatrix} x^{-1} & x^{-2} \\ -x^{-2} & -2x^{-3} \end{vmatrix} = -x^{-4}$$

(4)

$$W_1 = \begin{vmatrix} 0 & x^{-2} \\ \frac{\ln x}{x^2} & -2x^{-3} \end{vmatrix} = -x^{-4}\ln x$$

$$W_2 = \begin{vmatrix} x^{-1} & 0 \\ -x^{-2} & \frac{\ln x}{x^2} \end{vmatrix} = x^{-3}\ln x$$

Hence $u_1' = \dfrac{-x^{-4}\ln x}{-x^{-4}} = \ln x \Rightarrow u_1 = x\ln x - x$

and $u_2' = \dfrac{x^{-3}\ln x}{x^{-4}} = x\ln x \Rightarrow u_2 = \frac{1}{2}x^2\ln x + \frac{1}{4}x^2$

The particular soln is $y_p = \frac{1}{2}\ln x - \frac{3}{4}$ (Note: $y_p = u_1y_1 + u_2y_2$)

and the general soln is $y = c_1x^{-1} + c_2x^{-2} + \frac{1}{2}\ln x - \frac{3}{4}$

P 3

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Q3

$$y'' + 4y' + 4y = 3x + 4e^{x}$$

We have $y_c = c_1 e^{-2x} + c_2 x e^{-2x}$

and $y_p = Ax + B + Ce^{x}$

By Subs y_p in DE

$\Rightarrow$ $A = \frac{3}{4}$, $B = \frac{-3}{4}$ and $C = \frac{4}{9}$

$\therefore$ the general soln is

$$y = c_1 e^{-2x} + c_2 x e^{-2x} + \frac{3}{4}x - \frac{3}{4} + \frac{4}{9}e^{x}$$

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Q5

$x' + y'' = e^{3t} \Rightarrow Dx + D^2y = e^{3t}$ ①

$x' + y' + x - y = 4e^{3t} \Rightarrow (D+1)x + (D-1)y = 4e^{3t}$ ②

operat on eqn ② by D^2 and on eqn ① by $(D-1)$ and then Subtract

$[D^2(D+1) - D(D-1)]x = 36e^{3t} - 3e^{3t} + e^{3t}$

$\Rightarrow x''' + x' = 34e^{3t}$ ③

$\Rightarrow m^3 + m = 0$ characteristic eqn.

$\Rightarrow m(m^2+1) = 0 \Rightarrow m_1 = 0, \ m_2 = i, \ m_3 = -i$

$\Rightarrow x_c = c_1 + c_2 \cos t + c_3 \sin t$ ④

$\Rightarrow$ the particular soln is $x_p = Ae^{3t}$,

$\Rightarrow x_p' = 3Ae^{3t}, \ x_p'' = 9Ae^{3t}, \ x_p''' = 27Ae^{3t}$

Replacing in ③ we have

$27Ae^{3t} + 3Ae^{3t} = 34e^{3t}$

$\Rightarrow 30Ae^{3t} = 34e^{3t} \Rightarrow A = \frac{17}{15}$

$\therefore x_p = \frac{17}{15}e^{3t}$ ⑤

Thus a solution of the given system of DEs is

$x(t) = x_c + x_p$

④, ⑤ $\Rightarrow \quad = c_1 + c_2 \cos t + c_3 \sin t + \frac{17}{15}e^{3t}$

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