

P1

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Model answer of Mid II

55/1438

Q1 a) $a_2(x) = 4x^2 - 4$ is continuous $\forall x \in \mathbb{R}$

$a_1(x) = \sqrt{4x^2 - 4}$ N N $\forall x \in \mathbb{R}$ and $|x| \geq 1$

$a_0(x) = -4x\sqrt{4x^2 - 4}$ " " $\forall x \in [1, \infty)$

$\therefore a_2(x), a_1(x)$ and $a_0(x)$ are continuous on $[1, \infty)$,

(4) $a_2(x) = 0$ if $x = \pm 1$

$a_2(x) \neq 0 \forall x \in (1, \infty)$ or $(-\infty, -1)$

and $\therefore x_0 = 4 \in (1, \infty)$

\therefore the largest interval for which

the IVP has a unique solution is $(1, \infty)$



b) $x^2 y'' - xy' + 2y = 0$

$\Rightarrow y'' - \frac{1}{x}y' + \frac{2}{x^2}y = 0, x \neq 0$

$\Rightarrow p(x) = \frac{1}{x}$

$y_1(x) = x \sin(\ln x) \int \frac{e^{-\int \frac{1}{x} dx}}{x^2 \sin^2(\ln x)} dx$

(4) $y_2 = x \sin(\ln x) \int \frac{e^{\ln x}}{x^2 \sin^2(\ln x)} dx$

$y_2 = x \sin(\ln x) \int \csc^2(\ln x) \cdot \frac{1}{x} dx$

$y_2 = x \sin(\ln x) [-\cot(\ln x)]$

$y_2 = -x \cos(\ln x)$

\therefore the general solution of the given DE is

$y = C_1 x \sin(\ln x) + C_2 x \cos(\ln x)$

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P2
Q2

8

a) $y''' + 2y'' + y' = 5 - 46x + 3e^x + 5e^{-x}$

Ans:

we have

$$y_c = c_1 + c_2 e^{-x} + c_3 x e^{-x}$$

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and the particular soln has the form

$$y_p = Ax + Bx^2 + C \sin x + D e^x + E x^2 e^{-x}$$

b) $x^2 y'' + 4xy' + 2y = \ln x$, x70 cauchy Euler Eq.
For the homogeneous DE $x^2 y'' + 4xy' + 2y = 0$
we have $y_c = c_1 x^{-1} + c_2 x^{-2}$

To find the particular soln, we have

$$W = \begin{vmatrix} x^{-1} & x^{-2} \\ -x^{-2} & -2x^{-3} \end{vmatrix} = -x^{-4}$$

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$$W_1 = \begin{vmatrix} 0 & x^{-2} \\ \frac{\ln x}{x^2} & -2x^{-3} \end{vmatrix} = -x^{-4} \ln x$$

$$W_2 = \begin{vmatrix} x^{-1} & 0 \\ -x^{-2} & \frac{\ln x}{x^2} \end{vmatrix} = x^{-3} \ln x$$

Hence $u_1' = \frac{-x^{-4} \ln x}{-x^{-4}} = \ln x \Rightarrow u_1 = x \ln x - x$

and $u_2' = \frac{x^{-3} \ln x}{x^{-4}} = x \ln x \Rightarrow u_2 = \frac{1}{2} x^2 \ln x + \frac{1}{4} x^2$

The particular soln is $y_p = \frac{1}{2} \ln x - \frac{3}{4}$ Note $y_p = u_1 y_1 + u_2 y_2$

and the general soln is $y = c_1 x^{-1} + c_2 x^{-2} + \frac{1}{2} \ln x - \frac{3}{4}$

P 3

Q 3

4

$$y'' + 4y' + 4y = 3x + 4e^x$$

We have $y_c = c_1 e^{-2x} + c_2 x e^{-2x}$

and $y_p = Ax + B + Ce^x$

By Subs y_p in DE

$$\Rightarrow A = \frac{3}{4}, B = -\frac{3}{4} \text{ and } C = \frac{4}{9}$$

\therefore the general solⁿ is

$$y = c_1 e^{-2x} + c_2 x e^{-2x} + \frac{3}{4}x - \frac{3}{4} + \frac{4}{9}e^x$$

P4

Q5

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$$x' + y'' = e^{3t} \quad (1)$$

$$x' + y' + x - y = 4e^{3t} \Rightarrow (D+1)x + (D-1)y = 4e^{3t} \quad (2)$$

Operate on eqn (2) by D^2 and on eqn (1) by $(D-1)$ and then subtract

$$[D^2(D+1) - D(D-1)]x = 36e^{3t} - 3e^{3t} + e^{3t}$$

$$\Rightarrow x''' + x' = 34e^{3t} \quad (3)$$

$\Rightarrow m^3 + m = 0$ characteristic eqn.
 $\Rightarrow m(m^2 + 1) = 0 \Rightarrow m_1 = 0, m_2 = i, m_3 = -i$

$$\Rightarrow x_c = C_1 + C_2 \cos t + C_3 \sin t \quad (4)$$

\Rightarrow the particular soln is $x_p = Ae^{3t}$,
 $\Rightarrow x_p' = 3Ae^{3t}, x_p'' = 9Ae^{3t}, x_p''' = 27Ae^{3t}$

Replacing in (3) we have

$$27Ae^{3t} + 3Ae^{3t} = 34e^{3t}$$

$$\Rightarrow 30Ae^{3t} = 34e^{3t} \Rightarrow A = \frac{17}{15}$$

$$\therefore x_p = \frac{17}{15} e^{3t} \quad (5)$$

Thus a solution of the given system of DEs is

$$x(t) = x_c + x_p$$

$$(4), (5) \Rightarrow = C_1 + C_2 \cos t + C_3 \sin t + \frac{17}{15} e^{3t}$$

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