6. Here u and v denote the real and imaginary components of the function f defined by means of the equations

$$f(z)=\begin{cases}\dfrac{\overline{z}^2}{z} & \text{when } z\neq 0,\\ 0 & \text{when } z=0.\end{cases}$$

Now

$$f(z)=\underbrace{\frac{x^3-3xy^2}{x^2+y^2}}_{u}+i\underbrace{\frac{y^3-3x^2y}{x^2+y^2}}_{v}$$

when $z\neq 0$, and the following calculations show that

$$u_x(0,0)=v_y(0,0) \quad \text{and} \quad u_y(0,0)=-v_x(0,0):$$

$$u_x(0,0)=\lim_{\Delta x\to 0}\frac{u(0+\Delta x,0)-u(0,0)}{\Delta x}=\lim_{\Delta x\to 0}\frac{\Delta x}{\Delta x}=1,$$

$$u_y(0,0)=\lim_{\Delta y\to 0}\frac{u(0,0+\Delta y)-u(0,0)}{\Delta y}=\lim_{\Delta y\to 0}\frac{0}{\Delta y}=0,$$

$$v_x(0,0)=\lim_{\Delta x\to 0}\frac{v(0+\Delta x,0)-v(0,0)}{\Delta x}=\lim_{\Delta x\to 0}\frac{0}{\Delta x}=0,$$

$$v_y(0,0)=\lim_{\Delta y\to 0}\frac{v(0,0+\Delta y)-v(0,0)}{\Delta y}=\lim_{\Delta y\to 0}\frac{\Delta y}{\Delta y}=1.$$