**Exercise 1 :**

a) If $f = \ln(2x)$ and $g(x) = \int_1^{4x^2}(1+t^2)^{10}dt$. Since $f(\frac{1}{2}) = 0$ and $g(\frac{1}{2}) = 0$, then $F'(\frac{1}{2}) = 0$.

$(F'(x) = \frac{1}{x}\int_1^{4x^2}(1+t^2)^{10}dt + 8x\ln(2x)(1+16x^4)^{10}.)$

b)

$$\begin{aligned}\int_0^2 3x^2dx &= \lim_{n\to+\infty} 3\frac{2}{n}\sum_{k=1}^{n}\frac{4k^2}{n^2}\\ &= \lim_{n\to+\infty} 24\left(\frac{n(n+1)(2n+1)}{6n^3}\right) = 8.\end{aligned}$$

c) $f(x) = \sin^4(x)$

| $k$ | $x_k$ | $f(x_k)$ | $m$ | $mf(x_k)$ |
|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 0 |
| 1 | $\frac{\pi}{4}$ | $\frac{1}{4}$ | 2 | $\frac{1}{2}$ |
| 2 | $\frac{\pi}{2}$ | 1 | 2 | 2 |
| 3 | $\frac{3\pi}{4}$ | $\frac{1}{4}$ | 2 | $\frac{1}{2}$ |
| 4 | $\pi$ | 0 | 1 | 0 |
| | | | | 3 |

$\int_0^{\pi}\sin^4(x)dx \approx \frac{3\pi}{8}$.

**Exercise 2 :**

a) $f'(x) = \dfrac{1}{(\ln 2)(\sin^{-1}(x))\sqrt{1-x^2}}$.

b) $\displaystyle\int \frac{4^{-\ln(x)}}{x}dx \overset{t=-\ln(x)}{=} -\int 4^t dt = -\frac{4^{-\ln(x)}}{\ln 4} + c.$

c) $y = e^{2x^2\ln(x)}(x-1)^{\frac{3}{2}}$, $\ln(y) = 2x^2\ln(x) + \frac{3}{2}\ln(x-1)$.

$\dfrac{y'}{y} = 4x\ln(x) + 2x + \dfrac{3}{2(x-1)}$ and $y' = \left(4x\ln(x) + 2x + \dfrac{3}{2(x-1)}\right)y.$

**Exercise 3 :**

a)

$$\begin{aligned}\int \frac{2x+3}{\sqrt{4-x^2}}dx &= \int \frac{2x}{\sqrt{4-x^2}}dx + 3\int \frac{dx}{\sqrt{4-x^2}} \\ &= -2\sqrt{4-x^2} + 3\sin^{-1}(\frac{x}{2}) + c.\end{aligned}$$

b) $\displaystyle\int \frac{e^{\frac{x}{2}}}{7+e^x}dx \overset{t=e^{\frac{x}{2}}}{=} 2\int \frac{dt}{7+t^2} = \frac{2}{\sqrt{7}}\tan^{-1}(\frac{e^{\frac{x}{2}}}{7}) + c.$

c)

$$\begin{aligned}\int \frac{\sin(x)}{\sqrt{e^{\cos(x)}-1}}dx &\overset{t=\cos(x)}{=} -\int \frac{dt}{\sqrt{e^t-1}} \\ &\overset{u^2=e^t-1}{=} -\int \frac{2du}{1+u^2} \\ &= -2\tan^{-1}(\sqrt{e^{\cos(x)}-1}) + c.\end{aligned}$$

Or

$$\begin{aligned}\int \frac{\sin(x)}{\sqrt{e^{\cos(x)}-1}}dx &\overset{t=e^{\frac{1}{2}\cos(x)}}{=} -2\int \frac{dt}{t\sqrt{t^2-1}} \\ &= -2\sec^{-1}(e^{\frac{1}{2}\cos(x)}) + c.\end{aligned}$$