5.1)

6) $\int \frac{4}{z^7} - \frac{7}{z^4} + z \, dz = 4\frac{z^{-7+1}}{-7+1} - 7\frac{z^{-4+1}}{-4+1} + \frac{z^2}{2} + C =$

$\frac{4z^{-6}}{-6} + 7\frac{z^{-3}}{3} + \frac{z^2}{2} + C$

7) $\int 3u^{\frac{1}{2}} + \frac{1}{u^{\frac{1}{2}}} \, du = 3\frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1} + \frac{u^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + C = \frac{3u^{\frac{3}{2}}}{\frac{3}{2}} + 2u^{\frac{1}{2}} + C$

$= 2u^{\frac{3}{2}} + 2u^{\frac{1}{2}} + C$

12) $\int (x - \frac{1}{x})^2 dx = \int x^2 - 2 + \frac{1}{x^2} \, dx = \frac{x^3}{3} - 2x - \frac{1}{x} + C$

15) $\int (8x - 5) x^{-\frac{1}{3}} = \int 8x^{\frac{2}{3}} - 5x^{-\frac{1}{3}} \, dx = \frac{24x^{\frac{5}{3}}}{5} - \frac{15x^{\frac{2}{3}}}{2} + C$

18) $\int \frac{x^3 + 3x^2 - 9x - 2}{x - 2} \, dx$

By long division

$$\begin{array}{r|l} & \boxed{x^2 + 5x + 1} \\ x-2 & x^3 + 3x^2 - 9x - 2 \\ - & x^3 - 2x^2 \\ \hline & 5x^2 - 9x \\ - & 5x^2 - 10x \\ \hline & x - 2 \\ & x - 2 \\ \hline & 0 \end{array}$$

$\Rightarrow \int x^2 + 5x + 1 \, dx = \frac{x^3}{3} + \frac{5x^2}{2} + x + C$

20) $\int \frac{(\sqrt{t} + 2)^2}{t^3} dt = \int \frac{t + 4\sqrt{t} + 4}{t^3} dt = \int \frac{t}{t^3} + \frac{4\sqrt{t}}{t^3} + \frac{4}{t^3} dt$

$= \int \frac{1}{t^2} + 4t^{-\frac{5}{2}} + 4t^{-3} dt = -\frac{1}{t} - \frac{8}{3\sqrt{t^3}} - \frac{2}{t^2} + C$

24) $\int \frac{1}{4\sec x}\, dx = \frac{1}{4}\int \cos x\, dx = \frac{1}{4}\sin x + C$

27) $\int \frac{\sec t}{\cos t}\, dt = \int \sec t \left(\frac{1}{\cos t}\right) dt = \int \sec^2 t\, dt$

$= \tan t + C$

29) $\int \csc v \cot v \sec v\, dv = \int \frac{1}{\sin v} * \frac{\cancel{\cos v}}{\sin v} * \frac{1}{\cancel{\cos v}}\, dv$

$= \int \frac{1}{\sin^2 v}\, dv = \int \csc^2 v\, dv = -\cot v + C$

30) $\int 4 + 4\tan^2 v\, dv = \int 4(\underbrace{1+\tan^2 v}_{\sec^2 v})\, dv = 4\int \sec^2 v\, dv$

$= 4\tan v + C$

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5.2

21) $\int \frac{(\sqrt{x}+3)^4}{\sqrt{x}}\, dx$ let $u = \sqrt{x}+3$

$du = \frac{1}{2\sqrt{x}}\, dx \Rightarrow 2\,du = \frac{1}{\sqrt{x}}\, dx$

$\Rightarrow \int 2u^4\, du = 2\frac{u^5}{5} + C = \frac{2(\sqrt{x}+3)^5}{5} + C$

23) $\int \frac{t-2}{(t^2-4t+3)^3}\, dt \Rightarrow$ let $u = t^2-4t+3$

$du = 2t-4$

$du = 2(t-2)$

$\frac{1}{2}\, du = t-2\, dt$

$\Rightarrow \frac{1}{2}\int \frac{1}{u^3}\, du$

$= -\frac{1}{4u^2} + c = -\frac{1}{4(t^2-4t+3)^2} + c$

31) $\int \cos 3x \sqrt[3]{\sin 3x}\, dx$ let $u = \sin 3x$

$du = 3\cos 3x\, dx$

$\frac{1}{3}\, du = \cos 3x\, dx$

$\Rightarrow \frac{1}{3}\int \sqrt[3]{u}\, du = \frac{1}{\cancel{3}}\frac{u^{\frac{4}{3}}}{\frac{4}{\cancel{3}}}$

$= \frac{(\sin 3x)^{\frac{4}{3}}}{4} + C$

33) $\int (\sin x + \cos x)^2\, dx = \int \sin^2 x + 2\sin x \cos x + \cos^2 x\, dx$

$*\ \sin^2 x + \cos^2 x = 1$

$\sin 2x = 2\sin x \cos x$

$\Rightarrow \int 1 + \sin 2x\, dx = \int 1\, dx + \int \sin 2x\, dx = x + \int \sin 2x\, dx$

let $u = 2x$

$du = 2\, dx$

$\frac{1}{2}\, du = dx$

$\Rightarrow x + \frac{1}{2}\int \sin u\, du$

$= x - \frac{1}{2}\cos 2x + C$

34) $\int \frac{\sin 4x}{\cos 2x}\, dx = \int \frac{\sin 2(2x)}{\cos(2x)}$ $\quad \theta = 2x$

$\sin 2\theta = 2\sin\theta\cos\theta$

$\Rightarrow \int \frac{2\sin 2x \cancel{\cos 2x}}{\cancel{\cos 2x}}\, dx = \int 2\sin \underset{u}{2x}\, dx$

$= \int \sin u\, du = -\cos 2x + C$

35) $\int \sin x (1+\cos x)^2 dx$ $\quad$ let $u = 1+\cos x$

$du = -\sin x\, dx$

$-du = \sin x\, dx$

$-\int u^2 du = -\frac{u^3}{3} + C$

$= -\frac{(1+\cos x)^3}{3} + C$

37) $\int \frac{\sin x}{\cos^4 x}\, dx$ $\quad$ let $u = \cos x\, dx$ $\quad du = -\sin x\, dx$

$-du = \sin x\, dx$

$= -\int \frac{1}{u^4}\, du = +\frac{1}{3u^3} + C = \frac{1}{3\cos^3 x} + C$

38) $\int \sin 2x \sec^2 2x\, dx = \int \sin 2x \sec 2x \sec^4 2x\, dx$

$= \int \frac{\sin 2x}{\cos 2x} \sec^4 2x\, dx = \int (\tan 2x \sec 2x \sec^3 2x)$

$\Rightarrow \int \sec^3 2x (\tan 2x \sec(2x))\, dx$

$u = \sec 2x \quad du = 2\sec 2x \tan 2x\, dx$

$= \frac{1}{2}\int u^3 du = \frac{1}{2}\frac{u^4}{4} + C = \frac{(\sec 2x)^4}{8} + C$

39) $\int \frac{\cos t}{(1-\sin t)^2} dt \Rightarrow$ $u = 1 - \sin t$ $du = -\cos t \, dt$

$-du = \cos t \, dt$

$\Rightarrow \int -\frac{1}{u^2} du = +\frac{1}{u} + C = \frac{1}{1-\sin t} + C$

40) $\int (2+5\cos t)^3 \sin t \, dt$ $u = 2+5\cos t$

$du = -5 \sin t \, dt$

$-\frac{1}{5} du = \sin t \, dt$

$-\frac{1}{5}\int u^3 du$

$= -\frac{1}{5}\left(\frac{u^4}{4}\right) + C = -\frac{(2+5\cos t)^4}{20} + C$

41)

$\int \sec^2(3x-4) dx$ $u = 3x-4$

$du = 3 \, dx$

$\frac{1}{3} du = dx$

$\Rightarrow \frac{1}{3}\int \sec^2 u \, du$

$= \frac{1}{3}\tan(3x-4) + C$

42) $\int \frac{\csc 2x}{\sin 2x} dx = \int \csc^2 2x \, dx$ let $u = 2x$

$du = 2 \, dx$

$\frac{1}{2} du = dx$

$\Rightarrow \frac{1}{2}\int \csc^2 u \, du = -\frac{1}{2}\cot(2x) + C$

43) $\int \sec^2 3x \tan 3x \, dx = \int \sec 3x (\sec 3x \tan 3x) dx$

$u = \sec 3x$ $du = 3\sec 3x \tan 3x \, dx$

$\frac{1}{3}\int \cancel{\sec} u \, du = \frac{u^2}{6} + C = \frac{(\sec 3x)^2}{6} + C$

44) $\int \frac{1}{\tan 4x} * \frac{1}{\sin 4x} \, dx = \int \cot 4x \csc 4x \, dx$

$u = 4x \quad du = 4 \, dx$

$\frac{1}{4} \int \cot u \csc u \, du = \frac{1}{4} (-\csc(4x)) + C$

45) $\int \frac{1}{\sin^2 5x} \, dx \qquad u = 5x \quad du = 5 \, dx$

$\Rightarrow \frac{1}{5} \int \frac{1}{\sin^2 u} \, du = \frac{1}{5} \int \csc^2 u \, du = \frac{1}{5} (-\cot 5x) + C$

46) $\int \frac{x}{\cos^2(x^2)} \, dx \qquad u = x^2$

$du = 2x \, dx$

$\frac{1}{2} du = x \, dx$

$= \frac{1}{2} \int \sec^2(u) \, du = \frac{1}{2} \tan x^2 + C$

47) $\int x \cot(x^2) \csc(x^2) \, dx \qquad u = x^2$

$du = 2x \, dx$

$\frac{1}{2} \int \cot u \csc u \, du \qquad \frac{1}{2} du = dx$

$= \frac{1}{2} (-\cot x^2) + C$

48) $\int \sec\left(\frac{x}{3}\right) \tan\left(\frac{x}{3}\right) dx \qquad u = \frac{x}{3} \quad du = \frac{1}{3}$

$3 \, du = dx$

$\Rightarrow 3 \int \sec u \tan u \, du = 3 \sec \frac{x}{3} + C$