Business Statistics

QUA 207

Examples

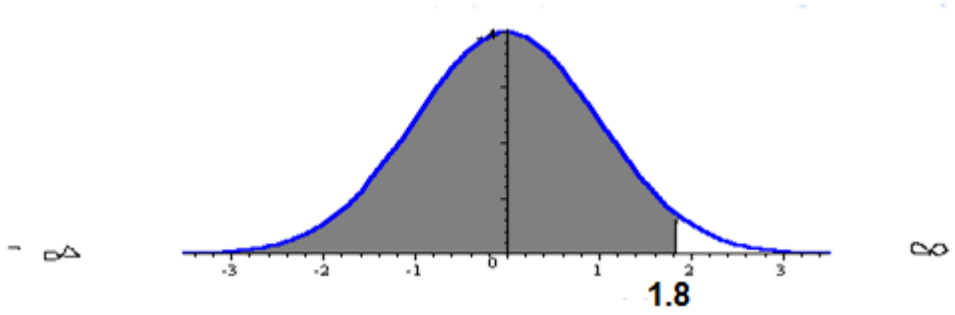
Examples Chapter (7)

Normal

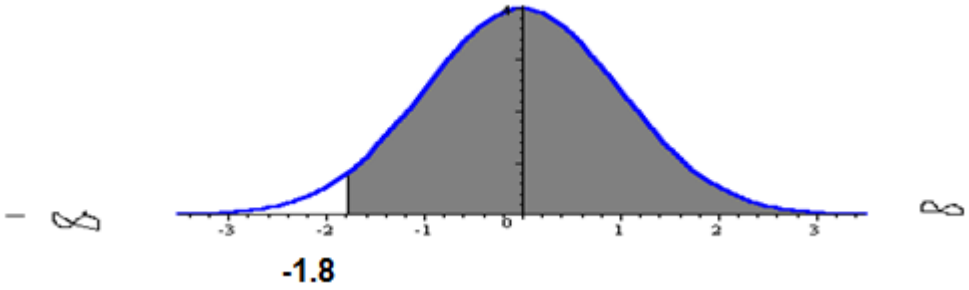
probability distribution

Example (1) : How to find the area under the normal curve?

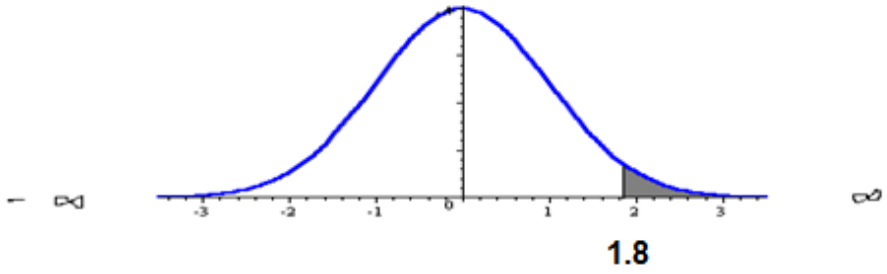
$$(1)\,P(Z < a) = 0.5 + p(0 \le Z \le a)$$
$$p(Z < 1.8) = 0.5 + P(0 < Z < 1.8)$$
$$= 0.5 + 0.4641 = 0.9641$$



$$(2)\,P(Z > -a) = 0.5 + P(-a \le Z \le 0)$$
$$p(Z > -1.8) = 0.5 + P(-1.8 < Z < 0)$$
$$= 0.5 + 0.4641 = 0.9641$$



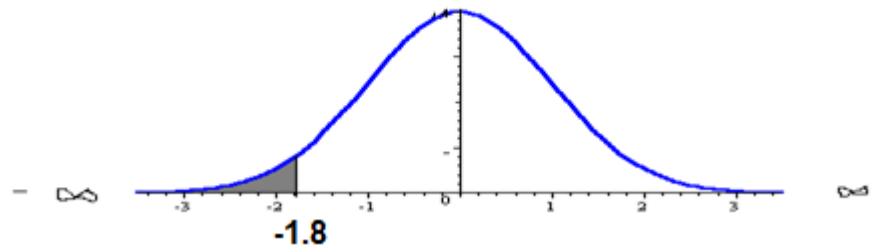
$$(3)\,P(Z > a) = 0.5 - P(0 \le Z \le 0)$$
$$p(Z > 1.8) = 0.5 - P(0 < Z < 1.8)$$
$$= 0.5 - 0.4641 = 0.0359$$



$(4) P(Z < -a) = 0.5 - P(-a \le Z \le 0)$

$p(Z < -1.8) = 0.5 - P(-1.8 < Z < 0)$

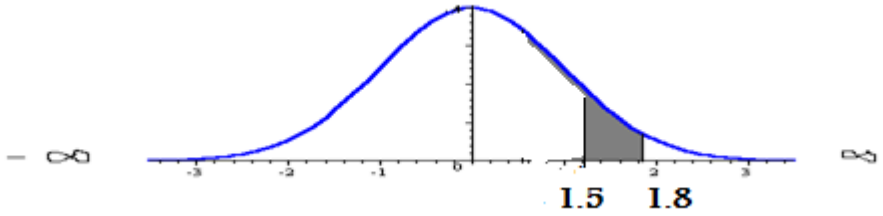
$= 0.5 - 0.4641 = 0.0359$



$(5) P(a < Z < b) = P(0 \le Z \le b) - P(0 \le Z \le a)$

$p(1.5 < Z < 1.8) = P(0 < Z < 1.8) - P(0 < Z < 1.5)$

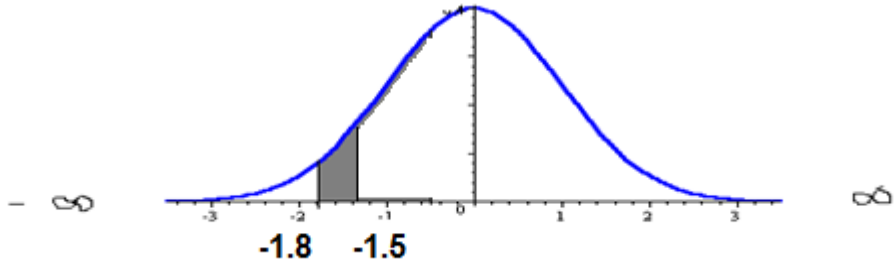
$= 0.4641 - 0.4332 = 0.0309$



$(6)\ P(-b < Z < -a) = P(-b \le Z \le 0) - P(-a \le Z \le 0)$

$p(-1.8 < Z < -1.5) = P(-1.8 < Z < 0) - P(-1.5 < Z < 0)$

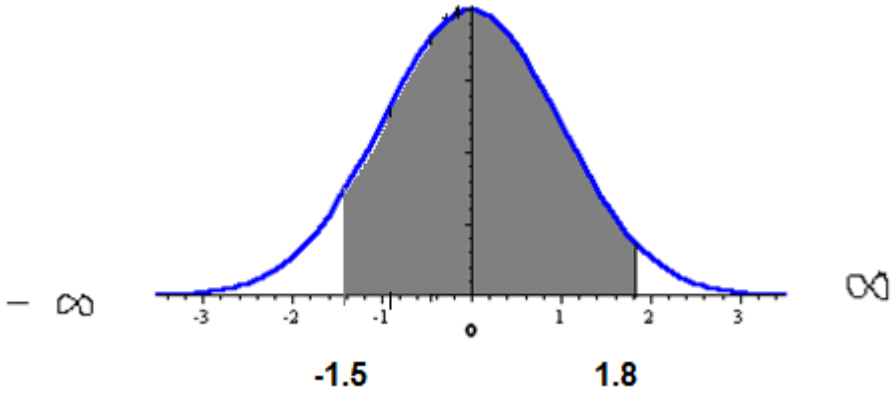
$= 0.4641 - 0.4332 = 0.0309$



$(7)\ P(-a < Z < b) = P(0 \le Z \le b) + P(-a \le Z \le 0)$

$p(-1.5 < Z < 1.8) = P(-1.5 < Z < 0) + P(0 < Z < 1.8)$

$= 0.4332 + 0.4641 = 0.8973$



Example (2)

Let X be a normally distributed random variable with mean 65 and standard deviation 13. Find the standard normal random variable (z) for P(X>80)

Solution:

$$P(X > 80) = P(Z > \frac{80-65}{13}) = P(Z > \frac{15}{13}) = P(Z > 1.15)$$

$$0.5 - 0.3749 = 0.1251$$

Example (3)

If the heights of 300 students are normally distributed, with a mean 172 centimeters and a standard deviation 8 centimeters, how many students have heights?

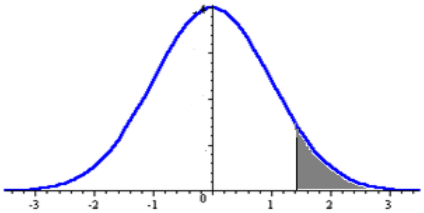
1. Greater than 184 centimeters.
2. Less than or equal to 160 centimeters.
3. Between 164 and 180 centimeters inclusive.
4. Equal to 172 centimeters.

$n = 300 \qquad \mu = 172 \qquad \sigma = 8$

$$P(x > 184) = P\left(Z > \frac{184-172}{8}\right) = P\left(Z > \frac{12}{8}\right)$$

1. $= P(z > 1.5) = 1 - \Phi(1.5) = 0.5 - 0.4332 = 0.0668$

   $number\ of\ students\ have\ heights\ greater\ than\ 184 = 300(0.0668) = 20$

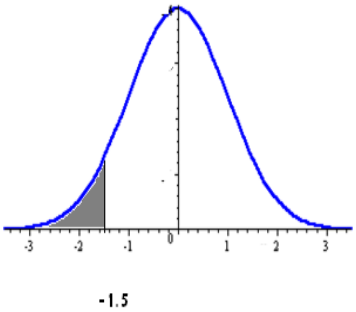


2.

$$P(x<160)=P\left(Z<\frac{160-172}{8}\right)=P\left(Z<\frac{-12}{8}\right)$$

$$=P(z<-1.5)=0.5-\Phi(1.5)=0.5-0.4332=0.0668$$

$$number\ of\ students\ have\ heights\ less\ than\ 160=300(0.0668)=20$$

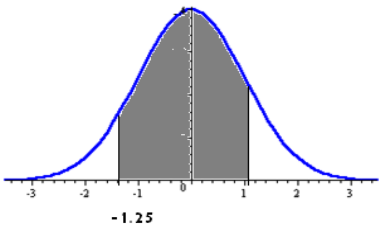


3.

$$P(164<x<180)=P\left(\frac{164-172}{8}<Z<\frac{180-172}{8}\right)=P\left(\frac{-8}{8}<Z<\frac{8}{8}\right)$$

$$=P(-1<z<1)=\Phi(1)+\Phi(1)=0.3413+0.3413=0.6826$$

$$number\ of\ students\ have\ heights\ between\ 164\ and\ 180=300(0.6826)=204.78\sim 205$$



4.

$$P(x=172)=0$$

Z-Table

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990