put $r = x^2+y^2 > 0$

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Solution 1st method

If u is radial harmonic function on the unit ball of $\mathbb{R}^2$ we can write

$$u(x,y) = g(x^2+y^2)$$

where $g : \mathbb{R}_+ \longrightarrow \mathbb{R}$

We have $\partial_x u(x,y) = 2x\, g'(x^2+y^2)$

and $\partial_{xx} u(x,y) = 4x^2 g''(x^2+y^2) + 2g'(x^2+y^2)$

so

$$\Delta u = 4(x^2+y^2)\, g'' + 4g'(x^2+y^2)$$

Take $h = g'$, then for $r>0$, we have: $r h'(r) + h(r) = 0$

so $(r h(r))' = 0$

$r h(r) = cst$ (constant)

. If we want $h \neq 0$ in order to have g non constant we take $h(r) = c/r$ with $c \neq 0$ and $r > 0$.

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But h is not defined at 0 and so g cannot be defined at 0

so $h \equiv 0$. Then

g is constant and u is constant

As $B(0)$ connected region.

Second method

if u is harmonic then $\exists f$ holomorphic function that satisfies $Re f = u = \varphi$...

and you can use Liouville Thm

and using the hypothesis that u is radial