Abstract
The quality and assessment of a reservoir can be documented in details by the application of number of moles. This research aims to calculate fractal dimension from the relationship among number of moles, maximum number of moles and wetting phase saturation and to confirm it by the fractal dimension derived from the relationship among capillary pressure and wetting phase saturation. In this research, porosity was measured on real collected sandstone samples and permeability was calculated theoretically from capillary pressure profile measured by mercury intrusion contaminating the pores of sandstone samples in consideration. Two equations for calculating the fractal dimensions have been employed. The first one describes the functional relationship between wetting phase saturation, number of moles, maximum number of moles and fractal dimension. The second equation implies to the wetting phase saturation as a function of capillary pressure and the fractal dimension. Two procedures for obtaining the fractal dimension have been utilized. The first procedure was done by plotting the logarithm of the ratio between number of mole and maximum number of moles versus logarithm wetting phase saturation. The slope of the first procedure = 3 - Df (fractal dimension). The second procedure for obtaining the fractal dimension was concluded by plotting the logarithm of capillary pressure versus the logarithm of wetting phase saturation. The slope of the second procedure = Df -3. On the basis of the obtained results of the fabricated stratigraphic column and the attained values of the fractal dimension, the sandstones of the Shajara reservoirs of the Shajara Formation were divided here into three units. The gained units from bottom to top are: Lower Shajara number of moles Fractal Dimension Unit, Middle Shajara number of moles Fractal Dimension Unit, and Upper Shajara number of moles Fractal Dimension Unit. The results show similarity between number of moles fractal dimension and capillary pressure fractal dimension. It was also noted that samples with wide range of pore radius were characterized by high values of fractal dimensions due to an increase in their connectivities which permit accommodation of high number of moles. In our case, and as conclusions the higher the fractal dimension, the higher the heterogeneity, the higher the permeability, the better the reservoir characteristics.

Keywords: Shajara Reservoirs, Shajara Formation, Number of moles fractal dimension

Introduction
The wetting phase saturation can be described as function of capillary pressure and fractal dimension was demonstrated by Toledo GT et al [1]. The Purcell model was found to be the best fit to the experimental data of the wetting phase relative permeability for the cases as long as the measured capillary pressure curve had the same residual saturation as the relative permeability curve was described by Li K and Horne RN [2]. A theoretical model to correlate capillary pressure and resistivity index based on the fractal scaling theory was reported by Li K and Williams W [3]. The fractal dimension resulting from longer transverse NMR relaxation times and lower capillary pressure reflects the volume dimension of larger pores was described by Zhang Z and Weller A [4]. The fractal dimension derived from the short NMR relaxation times is similar to the fractal dimension of the internal surface was described by Zhang Z and Weller A [4]. The fractal dimensions can be used to represent the complexity degree and heterogeneity of pore structure, and the coexistence of dissolution pores and large intergranular pores of Donghetang sandstones contributes to a heterogeneous pore throat distribution and a high value of fractal dimension was reported by Wang Z, et al. [5]. The relationship among capillary pressure (PC), nuclear magnetic transverse relaxation time (T2) and resistivity index (I) was studied by Guo Y-h, et al. [6]. An increase of bubble pressure fractal dimension and pressure head fractal dimension and decreasing pore size distribution index and fitting parameters m*n due to possibility of having interconnected channels was confirmed by AlKhidir KEME [7]. An increase of fractal dimension with increasing arithmetic, geometric relaxation time of induced polarization, permeability and grain size was investigated by AlKhidir KEME [8,9,10]. An increase of seismo electric and resistivity fractal dimensions with increasing permeability and grain size was described by AlKhidir KEME [11,12].

Materials and Methods
Sandstone samples were collected from the surface type section of

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the Permo-Carboniferous Shajara Formation, latitude 26°52′17.4″
longitude 43°36′18″. (Figure1). Porosity was measured on collected
samples using mercury intrusion Porosimetry and permeability was
derived from capillary pressure data. The purpose of this paper is
to obtain number of moles fractal dimension and to confirm it by
capillary pressure fractal dimension. The fractal dimension of the
first procedure is determined from the positive slope of the plot of
logarithm of the ratio of number of moles (NOM) to maximum
number of moles (NOM\text{max}) versus log wetting phase saturation
(logSw). Whereas the fractal dimension of the second procedure
is determined from the negative slope of the plot of logarithm of
log capillary pressure (log (Pc)) versus logarithm of wetting phase
saturation (logSw).

The number of moles can be scaled as

\[SW = \left[ \frac{NOM}{\text{max}} \right]^\frac{1}{1-Df} \]  

Where SW the water saturation, NOM the number of moles, Noma
the maximum number of moles, and Df the fractal dimension.
Equation 1 can be proofed from

\[ J = \text{CEK} \times \Delta P \]  

Where J the electric current density in ampere / square meter, CEK
the electro kinetic coefficient in ampere / (pascal*meter), and \Delta P
the pressure gradient in pascal /meter.
The electro kinetic coefficient can be scaled as

\[ \text{CEK} = Cs \times \sigma \]  

Where CEK the electro kinetic coefficient in ampere / (pascal *metre), Cs the streaming potential coefficient in volt / pascal, and \sigma
the fluid electric conductivity in Siemens /meter.
Insert equation 3 into equation 2

\[ J = Cs \times \sigma \times \Delta P \]  

The streaming potential coefficient can be scaled as

\[ Cs = \frac{\Psi}{\rho} \]  

Where Cs the streaming potential coefficient in volt / pascal, \Psi
the seismo electric transfer function in volt*square second /square
meter, \rho the density in kilo gram / cubic meter.
Insert equation 5 into equation 4

\[ J = \frac{\Psi \times \sigma \times \Delta P}{\rho} \]  

The density can be scaled as

\[ \rho = \left[ \frac{m}{V} \right] \]  

Where \rho the density in kilo gram /cubic meter, m the mass in kilo
gram, and V the volume in cubic meter.
Insert equation 7 into equation 6

\[ J \times m = \Psi \times \sigma \times \Delta P \times V \]  

The mass can be scaled as

\[ m = NOM \times mm \]  

Where m the mass in kilo gram, NOM the number of moles, and
mm the molecular mass in kilo gram / mole.
Insert equation 9 into equation 8

\[ J \times NOM \times mm = \Psi \times \sigma \times \Delta P \times V \]  

The electric conductivity can be scaled as

\[ \sigma = \frac{\text{reff}^2 \times CE}{8 \times \mu \times Cs} \]  

Where \sigma the electric conductivity of the fluid in Siemens /meter, reff
the effective pore radius in meter, CE the electro osmosis coefficient
in pascal / volt, \mu the fluid viscosity in pascal * second, and Cs the
streaming potential coefficient in volt / pascal.
Insert equation 11 into equation 10

\[ J \times NOM \times mm = \frac{\Psi \times \text{reff}^2 \times CE \times \Delta P \times V}{8 \times \mu \times Cs} \]  

When the pore radius is introduced equation 12 will become

\[ J \times NOM \times mm = \frac{\Psi r^2 \times CE \times \Delta P \times V}{8 \mu Cs} \]  

The maximum pore radius can be scaled as

\[ J \times NOM_{\text{max}} \times mm = \frac{\Psi r_{\text{max}}^2 \times CE \times \Delta P \times V}{8 \mu Cs} \]  

Divide equation 13 by equation 14

\[ \frac{J \times NOM \times mm}{J \times NOM_{\text{max}} \times mm} = \frac{\frac{\Psi r^2 \times CE \times \Delta P \times V}{8 \mu Cs}}{\frac{\Psi r_{\text{max}}^2 \times CE \times \Delta P \times V}{8 \mu Cs}} \]  

Equation 15 after simplification will become

\[ \frac{\frac{NOM}{NOM_{\text{max}}}}{\frac{1}{2}} = \left[ \frac{r^2}{r_{\text{max}}^2} \right] \]  

Take the square root of equation 16

\[ \sqrt{\frac{\frac{NOM}{NOM_{\text{max}}}}{\frac{1}{2}}} = \sqrt{\left[ \frac{r^2}{r_{\text{max}}^2} \right]} \]  

Equation 17 after simplification will become

\[ \frac{\frac{NOM}{NOM_{\text{max}}}}{\frac{1}{2}} = \frac{r}{r_{\text{max}}} \]  

Take the logarithm of equation 18

\[ \log \left[ \frac{\frac{NOM}{NOM_{\text{max}}}}{\frac{1}{2}} \right] = \log \left[ \frac{r}{r_{\text{max}}} \right] \]
But, \( \log \left( \frac{r}{r_{\text{max}}} \right) = \frac{\log S_w}{[3 - D_f]} \)  

Insert equation 20 into equation 19

\[ \log \frac{S_w}{[3 - D_f]} = \log \left( \frac{\frac{1}{N_{\text{OM}}} \left( \frac{S_w}{[3 - D_f]} \right)^{\frac{1}{3}}}{S_w} \right) \]

Equation 21 after log removal will become

\[ S_w = \left( \frac{N_{\text{OM}}}{N_{\text{OM, max}}} \right)^{\left( \frac{1}{3} - D_f \right)} \]

Equation 22 the proof of equation 1 which relates the water saturation, the number of moles, the maximum number of moles, and the fractal dimension. The capillary pressure can be scaled as

\[ \log S_w = [D_f - 3] * \log p_c + \text{constant} \]

Where \( S_w \) the water saturation, \( D_f \) the fractal dimension, and \( p_c \) the capillary pressure.

**Result and discussion**

Based on field observation the Shajara Reservoirs of the Permo-Carboniferous Shajara Formation were divided here into three units as described in Figure 1. These units from bottom to top are: Lower Shajara Reservoir, Middle Shajara reservoir, and Upper Shajara Reservoir. Their acquired results of the number of moles fractal dimension and capillary pressure fractal dimension are displayed in Table 1. Based on the attained results it was found that the number of moles fractal dimension is equal to the capillary pressure fractal dimension. The maximum value of the fractal dimension was found to be 2.7872 assigned to sample SJ13 from the Upper Shajara Reservoir as verified in Table 1. Whereas the minimum value of the fractal dimension 2.4379 was reported from sample SJ3 from the Lower Shajara reservoir as displayed in Table 1. The number of moles fractal dimension and capillary pressure fractal dimension were observed to increase with increasing permeability as proofed in Table 1 owing to the possibility of having interconnected channels.

The Lower Shajara reservoir was denoted by six sandstone samples (Figure 1), four of which label as SJ1, SJ2, SJ3 and SJ4 were selected for capillary pressure measurement as confirmed in Table 1. Their positive slopes of the first procedure log of the ratio of number of moles (NOM) to maximum number of moles (NOM\(_{\text{max}}\)) versus log wetting phase saturation (Sw) and negative slopes of the second procedure log capillary pressure (Pc) versus log wetting phase saturation (Sw) are delineated in Figure 2, Figure 3, Figure 4, and Figure 5. Their number of moles fractal dimension and capillary pressure fractal dimension values are shown in Table 1. As we proceed from sample SJ2 to SJ3 a pronounced reduction in permeability due to compaction was reported from 1955 md to 56 md which reflects decrease in number of moles fractal dimension from 2.7748 to 2.4379 as specified in table 1. Again, an increase in grain size and permeability was verified from sample SJ4 whose number of moles fractal dimension and capillary pressure fractal dimension was found to be 2.6843 as described in Table 1.

<table>
<thead>
<tr>
<th>Formation</th>
<th>Reservoir</th>
<th>Sample</th>
<th>Porosity %</th>
<th>k (md)</th>
<th>Positive slope of the first procedure Slope=3-Df</th>
<th>Negative slope of the second procedure Slope=Df-3</th>
<th>Number of moles fractal dimension</th>
<th>Capillary pressure fractal dimension</th>
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<td>Upper Shajara</td>
<td>SJ13</td>
<td>25</td>
<td>973</td>
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<td></td>
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<td></td>
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<td>36</td>
<td>1197</td>
<td>0.2414</td>
<td>-0.2414</td>
<td>2.7586</td>
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<td></td>
<td>Middle Shajara</td>
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<td>1394</td>
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<tr>
<td></td>
<td></td>
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<td></td>
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<td></td>
<td></td>
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<td>35</td>
<td>1955</td>
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<td></td>
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<td>0.2141</td>
<td>-0.2141</td>
<td>2.7859</td>
<td>2.7859</td>
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</table>
Figure 1: Surface type section of the Shajara Reservoirs of the Permo-Carboniferous Shajara formation at latitude 26° 52’ 17.4”, longitude 43° 36’ 18’.

Figure 2: Log (NOM_{1/2}/NOM_{1/2, max}) & log Pc versus log Sw of sample SJ1

Figure 3: Log (NOM_{1/2}/NOM_{1/2, max}) & log Pc versus log Sw of sample SJ2

Figure 4: Log (NOM_{1/2}/NOM_{1/2, max}) & log Pc versus log Sw of sample SJ3

Figure 5: Log (NOM_{1/2}/NOM_{1/2, max}) & log Pc versus log Sw of sample SJ4

In contrast, the Middle Shajara reservoir which is separated from the Lower Shajara reservoir by an unconformity surface as shown in Figure 1. It was designated by four samples (Figure 1), three of which named as SJ7, SJ8, and SJ9 as illustrated in Table 1 were selected for capillary measurements as described in Table 1. Their positive slopes of the first procedure and negative slopes of the second procedure are shown in Figure 6, Figure 7 and Figure 8 and Table 1. Additionally, their number of moles fractal dimensions and capillary pressure fractal dimensions show similarities as delineated in Table 1. Their fractal dimensions are higher than those of samples SJ3 and SJ4 from the Lower Shajara Reservoir due to an increase in their permeability as explained in Table 1. On the other hand, the Upper Shajara reservoir was separated from the Middle Shajara reservoir by yellow green mudstone as revealed in Figure 1. It is defined by three samples so called SJ11, SJ12, SJ13 as explained in Table 1. Their positive slopes of the first procedure and negative slopes of the second procedure are displayed in Figure 9, Figure 10 and Figure 11 and Table 1. Moreover, their number of moles fractal dimension and capillary pressure fractal dimension are also higher than those of sample SJ3 and SJ4 from the Lower Shajara Reservoir due to an increase in their permeability as clarified in Table 1.

Overall a plot of number of moles fractal dimension versus capillary pressure fractal dimension as shown in Figure 12 reveals three permeable zones of varying Petrophysical properties. Such variation in fractal dimension can account for heterogeneity which is a key parameter in reservoir quality assessment. This reservoir heterogeneity was also confirmed by plotting positive slope of the first procedure versus negative slope of the second procedure as described in Figure 13.
Overall a plot of number of moles fractal dimension versus capillary pressure fractal dimension as shown in Figure 12 reveals three permeable zones of varying Petrophysical properties. Such variation in fractal dimension can account for heterogeneity which is a key parameter in reservoir quality assessment. This reservoir heterogeneity was also confirmed by plotting positive slope of the first procedure versus negative slope of the second procedure as described in Figure 13.
Conclusion

• The sandstones of the Shajara Reservoirs of the Shajara formation permo-Carboniferous were divided here into three units based on number of moles fractal dimension.

• The Units from base to top are: Lower Shajara number of molesFractal dimension Unit, Middle Shajara number of molesFractal Dimension Unit, and Upper Shajara number of molesFractal Dimension Unit.

• These units were also proved by capillary pressure fractal dimension.

• The fractal dimension was found to increase with increasing grain size and permeability.

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References


