Examples Chapter(10)
One-Sample Tests of Hypothesis

P-VALUE

The p-value or observed of significance level of a statistical test is the smallest value of α for which H_0 can be rejected. It is the actual risk of committing a type I error, if H_0 is rejected based on the observed value of the test statistic. The p-value measures the strength of the evidence against H_0.

The probability of observing samples data by chance under the Null hypothesis (i.e. null hypothesis is true).

In testing a hypothesis, we can also compare the *p*-value to with the significance level (α).

- **If the *p*-value < significance level(α) , H_0 is rejected (means significant result)**
- **If the *p*-value $\geq$ significance level (α), H_0 is not rejected. (means not significant result).**

Compute the P- value (Only we used the Z distribution)

Case 1 : If the one – tailed test (Right)

$$p-value = P(Z > z_c) = 0.5 - \Phi(z_c)$$

Case 2: If the one – tailed test (left)

$$p-value = P(Z < -z_c) = 0.5 - \Phi(z_c)$$

Case3: If the two – tailed test

$$p-value = P(Z > z_c) + P(Z < -z_c) = [0.5 - \Phi(z_c)] + [0.5 - \Phi(z_c)] = 1 - 2\,\Phi(z_c)$$

Or

$$p-value = 2P(Z > |z_c|) = 2[0.5 - \Phi(z_c)]$$

Example

Refer to Jamestown Steel Company example (Slide 8 / chapter 9) compute the p-Value .

$H_0 : \mu = 200 \quad H_1 : \mu \neq 200$

$\mu_0 = 200 \quad , \quad \overline{X} = 203.5 \quad , \quad \sigma = 16 \quad , \quad n = 50 \quad , \alpha = 0.01$

$$p-value = 2P(Z > |z_c|) = 2[0.5 - \Phi(z_c)]$$

$$= 2P(Z > 1.55) = 2[0.5 - \Phi(1.55)] = 2[0.5 - 0.4394] = 2 \times 0.0606 = 0.1212$$

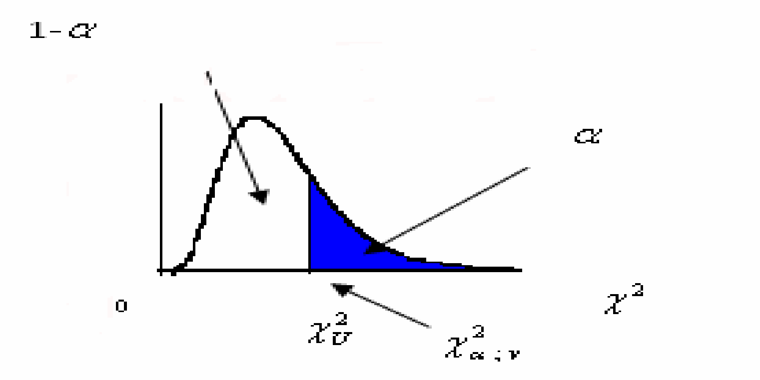
$P-value = 0.1212 > \alpha = 0.01$

$\therefore Do \;\; not \;\; reject \; H_0$

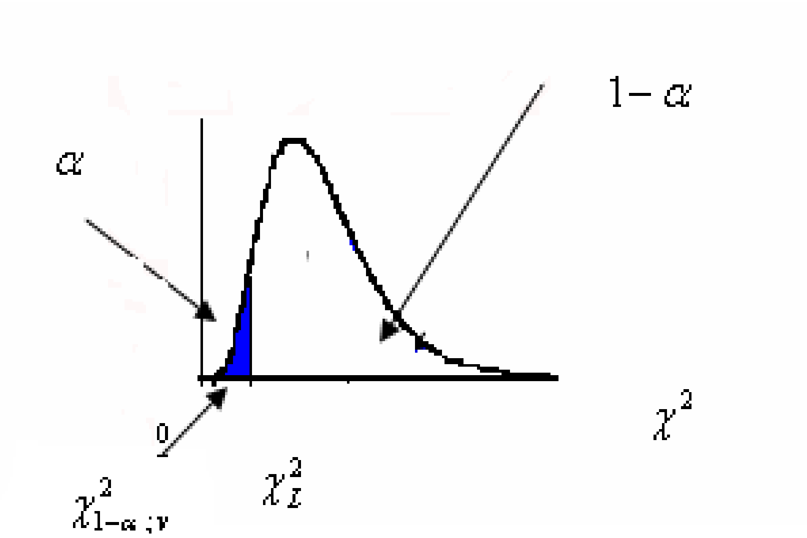
Test of hypothesis concerning apopulation variance

Step (1) : State the Null (H_0) and alternate (H_1) hypothesis

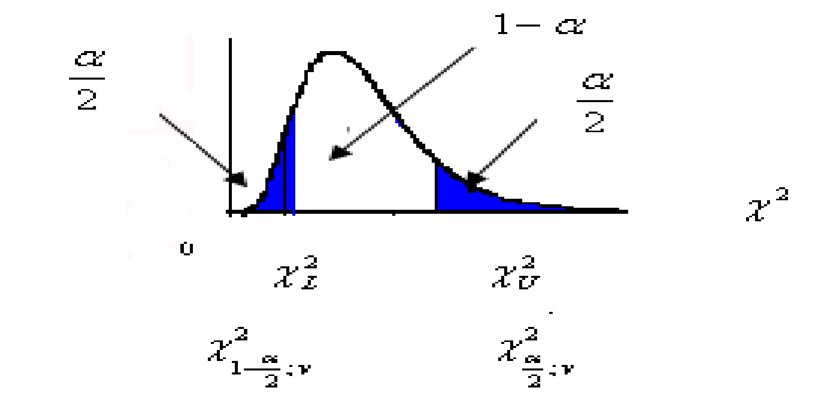
Case 1: $H_0 : \sigma^2 \le \sigma_0^2$
$H_1 : \sigma^2 > \sigma_0^2$



Case 2: $H_0 : \sigma^2 \ge \sigma_0^2$
$H_1 : \sigma^2 < \sigma_0^2$



Case 3: $H_0 : \sigma^2 = \sigma_0^2$
$H_1 : \sigma^2 \ne \sigma_0^2$



Step (2): Select a level of significance

Step (3): Select the Test Statistic (computed value)

$$\chi^2 = \frac{(n-1)S^2}{\sigma^2}$$

Step (4): Selected the Critical value

The one – tailed test (Right)	$\chi^2_{(\alpha,n-1)}$
The one – tailed test (left)	$\chi^2_{(1-\alpha,n-1)}$
The two – tailed test	$\chi^2_{\left(\frac{\alpha}{2},n-1\right)} \& \chi^2_{\left(1-\frac{\alpha}{2},n-1\right)}$

Step (5): Formulate the Decision Rule and Make a Decision

Case1: The one – tailed test (Right)

Reject H_0 if $\boldsymbol{\chi^2_c} > \chi^2_{(\alpha,n-1)}$

Case2: The one – tailed test (left)

Reject H_0 if $\boldsymbol{\chi^2_c < \chi^2_{(1-\alpha,n-1)}}$

Case3: The two – tailed test; rejectH_0 if

$\boldsymbol{\chi^2_c > \chi^2_{\left(\frac{\alpha}{2},n-1\right)}}$ Or $\boldsymbol{\chi^2_c < \chi^2_{\left(1-\frac{\alpha}{2},n-1\right)}}$

Example (6)

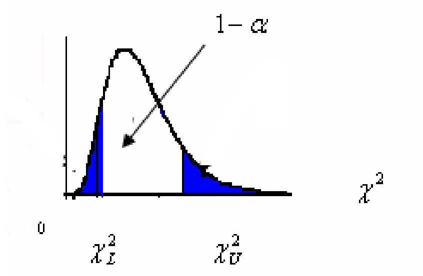
A sample of size 10 produced a variance of 14.Is this sufficient to reject the null hypothesis that σ^2 is equal to 6

When tested using a 0.05 level of significance?

Step 1: State the null hypothesis and the alternate hypothesis.

$H_0 : \sigma^2 = 6$

$H_1 : \sigma^2 \neq 6$



This is two-tailed test

(Note: keyword in the problem "that σ^2 is equal to 6")

Step 2: Select the level of significance.

α = 0.05 as stated in the problem

$$\frac{\alpha}{2} = \frac{0.05}{2} = 0.025$$

$$1 - \frac{\alpha}{2} = 1 - \frac{0.05}{2} = 0.975$$

Step 3: Select the test statistic.

$$\chi_c^2 = \frac{(n-1)S^2}{\sigma^2} = \frac{(10-1)14}{6} = \frac{126}{6} = 21$$

Step 4: Formulate the decision rule (Critical value)

$\chi^2_{0.025\ ;.9} = 19.022$, $\chi_{0.975\ ;\ 9} = 2.7003$

Right tail areas for the Chi-square Distribution					
V	Q				
df\area	0.250	0.100	0.050	0.025	0.010
1	1.3233	2.7055	3.8415	5.0239	6.6349
2	2.7726	4.6052	5.9915	7.3778	9.2104
3	4.1083	6.2514	7.8147	9.3484	11.3449
4	5.3853	7.7794	9.4877	11.1433	13.2767
5	6.6257	9.2363	11.0705	12.8325	15.0863
6	7.8408	10.6446	12.5916	14.4494	16.8119
7	9.0371	12.0170	14.0671	16.0128	18.4753
8	10.2189	13.3616	15.5073	17.5345	20.0902
9	11.3887	14.6837	16.9190	19.0228	21.6660
10	12.5489	15.9872	18.3070	20.4832	23.2093

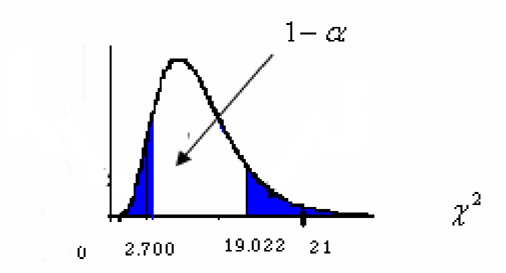
Right tail areas for the Chi-square Distribution					
V dflarea	Q				
	0.750	0.900	0.950	0.975	0.990
1	0.101531	0.015791	0.003932	0.000982	0.000157
2	0.575364	0.210721	0.102586	0.050636	0.020100
3	1.212532	0.584375	0.351846	0.215795	0.114832
4	1.922568	1.063624	0.710724	0.484419	0.297107
5	2.674604	1.610309	1.145477	0.831209	0.554297
6	3.454598	2.204130	1.635380	1.237342	0.872083
7	4.254852	2.833105	2.167349	1.689864	1.239032
8	5.070642	3.489537	2.732633	2.179725	1.646506
9	5.898823	4.168156	3.325115	2.700389	2.087889
10	6.737199	4.865178	3.940295	3.246963	2.558199
11	7.584145	5.577788	4.574809	3.815742	3.053496

Step 5: Make a decision and interpret the result.

Reject H_0 if $\chi_c^2 > \chi^2_{\frac{\alpha}{2},v} = \chi^2_{0.025\ ;.9} = 19.022$,

Or $\chi_c^2 < \chi^2_{1-\frac{\alpha}{2},v} = \chi_{0.975\ ;\ 9} = 2.7003$

The decision is to reject the null hypothesis, because the computed χ^2 Value (21) is larger than the critical value (19.022).
We conclude that there is a difference



The Types of errors

Null Hypothesis	**Does Not RejectH_0**	**Rejects H_0**
H_0 **is true**	**Do not rejecting The null hypothesis,** H_0 **,when It is true(**$1-\alpha$**)** **{Correct decision}**	**rejecting The null hypothesis,** H_0 **,when It is true(**α**)** **{Type I error}**
H_0 **is false**	**Do not rejecting The null hypothesis,** H_0 **,when It is false(**β**)** **{Type II error}**	**rejecting The null hypothesis,** H_0 **,when It is false (Power) (**$1-\beta$**)** **{Correct decision}**

Note:

The quantity $(1-\beta)$ is called the power of the test because it measures the probability of taking the action that we wish to have occur-that is ,rejecting the H0 when it is false and H1 is true.

$(1-\beta)$ = P(rejecting the H0 when it is false)

Ideally, you would like (α) to be small and the power $(1-\beta)$ to be large.

Example :

A manufacturer purchases steel bars to make cotter pins. Past experience indicates that the mean tensile strength of all incoming shipments is greater than 10,000 psi and that the standard deviation, σ, is 400 psi. In order to make a decision about incoming shipments of steel bars, the manufacturer set up this rule for the quality-control inspector to follow: “Take a sample of 100 steel bars, at the .05 significance level.

Suppose the unknown population mean of an incoming lot, designated μ_1 is really 10120 psi. Find

a .The type I error (Rejecting the null hypothesis, H_0, when It is true (α)).

b. The correct decision (Do not rejecting the null hypothesis, H_0, when It is true ($1-\alpha$).

c .The type II error (Do not rejecting the null hypothesis, H_0, when It is false (β)).

d .The correct decision (Rejecting The null hypothesis, H_0, when It is false ($1-\beta$)).

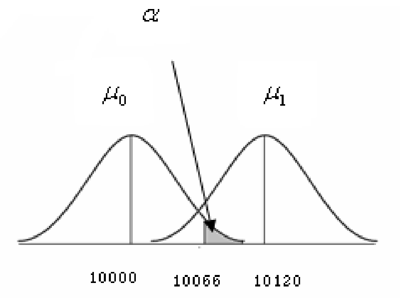
Solution:

$H_0: \mu \le 10000$
$H_1: \mu > 10000$ $Z = 1.645$

$$\mu_0 + Z_\alpha \frac{\sigma}{\sqrt{n}} = 10000 + 1.645\frac{400}{10} = 10000 + 65.8 = 10066$$

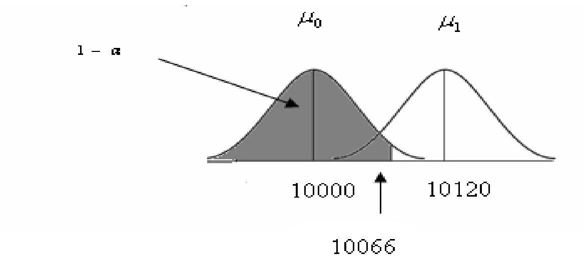
a .The type I error (where rejecting the null hypothesis, H_0, when it is true (α)).

$\alpha = 0.05$



b. The correct decision (Do not rejecting The null hypothesis, H_0, when It is true ($1-\alpha$).

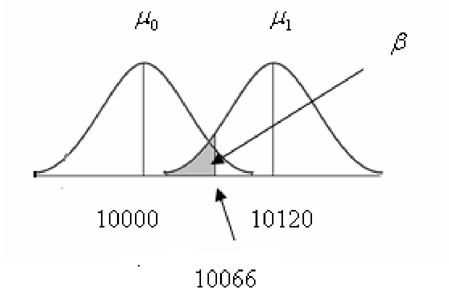
$1-\alpha = 1-.05 = 0.95$



c .The type II error (where accepting the null hypothesis, H_0, when it is false (β)).

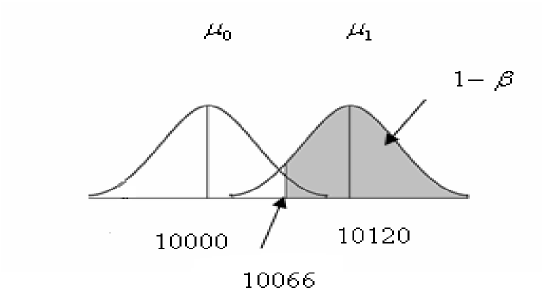
$$P\left(Z < \frac{\overline{X}_c - \mu_1}{\frac{\sigma}{\sqrt{n}}}\right) = P\left(Z < \frac{10066 - 10120}{\frac{400}{\sqrt{100}}}\right) = P\left(Z < \frac{-54}{40}\right) = 0.5 - P(-1.35 < Z < 0)$$

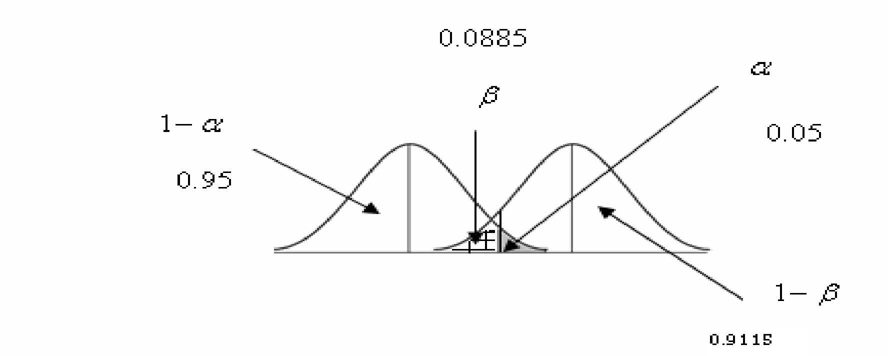
$0.5 - \Phi(1.35) = 0.5 - 0.4115 = 0.0885 \therefore \beta = 0.0885$



d .The correct decision (where H_0 is false and reject it ($1-\beta$)).

$1-\beta = 1-0.0885 = 0.9115$





Example :

A manufacturer purchases steel bars to make cotter pins. Past experience indicates that the mean tensile strength of all incoming shipments is less than 10,000 psi and that the standard deviation, σ, is 400 psi. In order to make a decision about incoming shipments of steel bars, the manufacturer set up this rule for the quality-control inspector to follow: "Take a sample of 100 steel bars, at the .05 significance level.

Suppose the unknown population mean of an incoming lot, designated μ_1 is really 9900psi.find:

a .The type I error (Rejecting the null hypothesis, H_0, when It is true (α)).

b. The correct decision (Do not rejecting the null hypothesis, H_0, when It is true ($1-\alpha$)).

c .The type II error (Do not rejecting the null hypothesis, H_0, when It is false (β)).

d .The correct decision (Rejecting the null hypothesis, H_0, when It is false ($1-\beta$)).

Solution:

$H_0 : \mu \geq 10000$

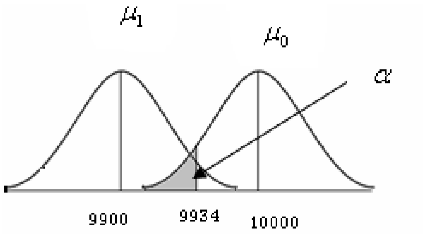
$H_1 : \mu. < 10000$

$Z = 1.645$

$$\mu_0 - Z_\alpha \frac{\sigma}{\sqrt{n}} = 10000 - 1.645\frac{400}{10} = 10000 - 65.8 = 9934$$

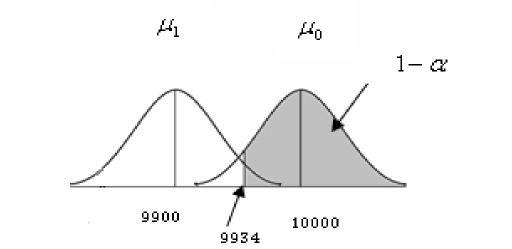
a .The type I error (where rejecting the null hypothesis, H_0, when it is true (α)).

$\alpha = 0.05$



b. The correct decision (Do not rejecting the null hypothesis, H_0, when It is true $(1-\alpha)$).

$1-\alpha = 1-0.05 = 0.95$



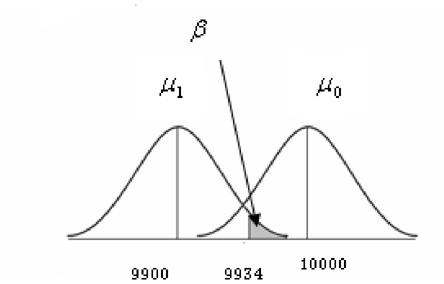
c .The type II error (where accepting the null hypothesis, H_0, when it is false (β)).

$$P\left(Z > \frac{\overline{X}_c - \mu_1}{\frac{\sigma}{\sqrt{n}}}\right)$$

$$= P\left(Z > \frac{9934-9900}{\frac{400}{\sqrt{100}}}\right) = P\left(Z > \frac{34}{40}\right) = 0.5 - P(0 < Z < 0.85)$$

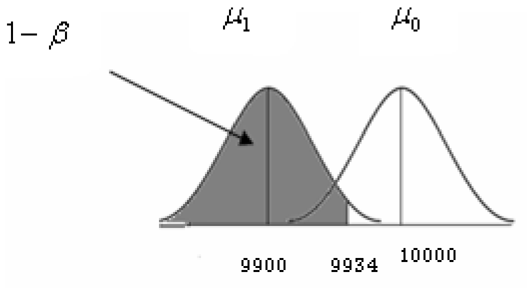
$0.5 - \Phi(0.85) = 0.5 - 0.3023 = 0.1977$

$\therefore \beta = 0.1977$



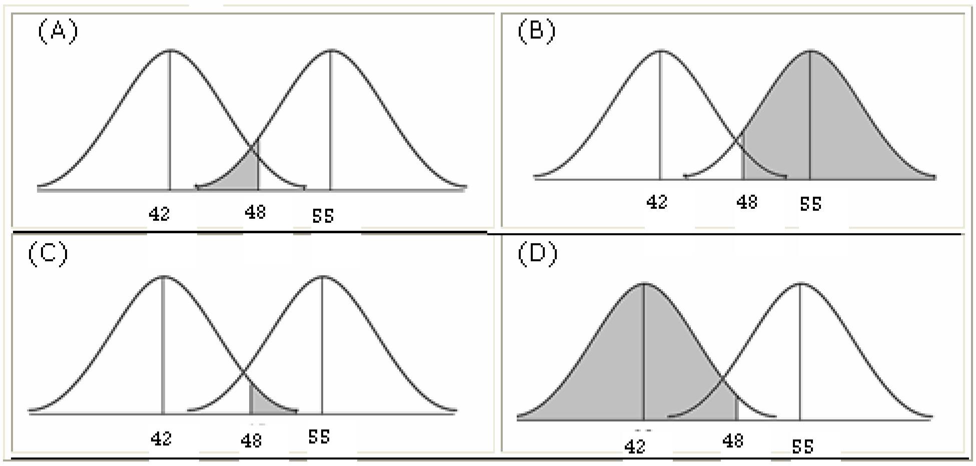
d .The correct decision (where H_0 is false and reject it $(1-\beta)$).

$1-\beta = 1-0.1977 = 0.8023$



Extra examples
Example
If

$\mu_1 = 55$, $H_0 : \mu \le 42$
$H_1 : \mu > 42$



Complete the following statements:

a. The probability of not rejecting the null hypothesis when it is true is the shaded area in diagram..........

b. The probability of Type I error is the shaded area in diagram…….

c. The probability of Type II error is the shaded area in diagram…….

d .The probability of rejecting the null hypothesis when it is false is the shaded area in. diagram..........

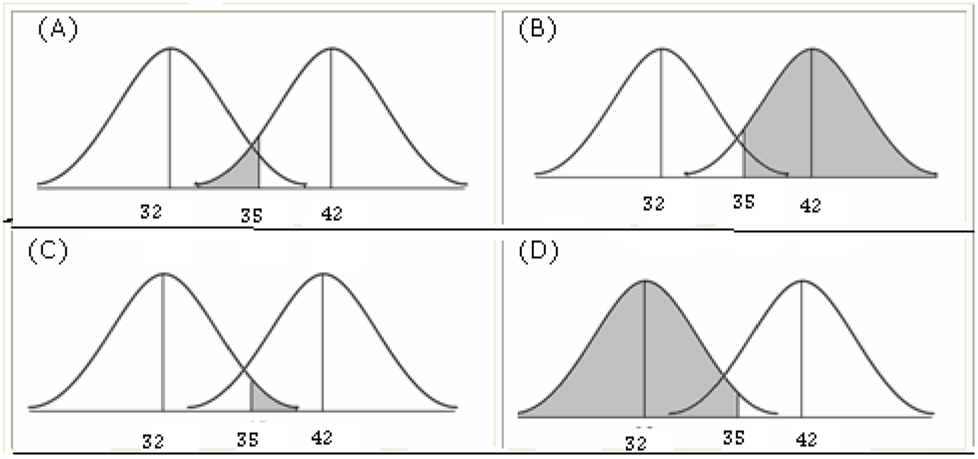
Solution:

a. Diagram D
b. Diagram C
c. Diagram A
d. Diagram B

Example

If

$\mu_1 = 32$, $H_0 : \mu \geq 42$
$H_1 : \mu < 42$



Complete the following statements:

a. The probability of not rejecting the null hypothesis when it is true is the shaded area in diagram..........

b. The probability of Type I error is the shaded area in diagram…….

c. The probability of Type II error is the shaded area in diagram…….

d .The probability of rejecting the null hypothesis when it is false is the shaded area in diagram..........

Solution:

a. Diagram B
b. Diagram A
c. Diagram C
d. Diagram D

Example

Null Hypothesis	Does Not Reject H_0	Rejects H_0
H_0 is true	Do not rejecting The null hypothesis, H_0 ,when It is true($1-\alpha$) Example √ $H_0 : \mu \geq 60$ x $H_1 : \mu < 60$ If the decision is Do not reject H_0 Let $\mu_1 = 80$ $\therefore$ The decision is correct Do not rejecting The null hypothesis, H_0 ,when It is true($1-\alpha$)	rejecting The null hypothesis, H_0 ,when It is true(α) Example x $H_0 : \mu \geq 60$ √ $H_1 : \mu < 60$ If the decision is reject H_0 Let $\mu_1 = 80$ $\therefore$ The decision is incorrect rejecting The null hypothesis, H_0 ,when It is true(α)
H_0 is false	Do not rejecting The null hypothesis, H_0 ,when It is false(β) Example √ $H_0 : \mu \geq 60$ x $H_1 : \mu < 60$ If the decision is Do not reject H_0 Let $\mu_1 = 50$ $\therefore$ The decision is incorrect Do not rejecting The null hypothesis, H_0 ,when It is false(β)	rejecting The null hypothesis, H_0 ,when It is false (Power)($1-\beta$) Example x $H_0 : \mu \geq 60$ √ $H_1 : \mu < 60$ If the decision is reject H_0 Let $\mu_1 = 50$ $\therefore$ The decision is correct rejecting The null hypothesis, H_0 ,when It is false (Power)($1-\beta$)

Note about P_c :

$H_0 : \pi \le \pi_0$ $H_1 : \pi > \pi_0$,	$\pi_0 + Z_\alpha \sqrt{\dfrac{\pi_0(1-\pi_0)}{n}}$,	$\beta = P\left(Z < \dfrac{P_c - \pi_1}{\sqrt{\dfrac{\pi_1(1-\pi_1)}{n}}} \right)$

$H_0 : \pi \ge \pi_0$ $H_1 : \pi < \pi_0$,	$\pi_0 - Z_\alpha \sqrt{\dfrac{\pi_0(1-\pi_0)}{n}}$,	$\beta = P\left(Z > \dfrac{P_c - \pi_1}{\sqrt{\dfrac{\pi_1(1-\pi_1)}{n}}} \right)$