

$$\langle u, v \rangle = 0$$

$$\Rightarrow \langle au, bv \rangle = 0 \forall a, b \in \mathbb{R}$$

Proof :

$$\langle au, bv \rangle = ab \langle u, v \rangle$$

$$= ab(0) = 0$$

Example 8 – Page 372

$$\mathbf{v}_1 = \mathbf{u}_1 = (1, 1, 1)$$

$$\mathbf{v}_2 = \mathbf{u}_2 - \frac{\langle \mathbf{u}_2, \mathbf{v}_1 \rangle}{\|\mathbf{v}_1\|^2} \mathbf{v}_1 = \left(-\frac{2}{3}, \frac{1}{3}, \frac{1}{3} \right)$$

$$\mathbf{v}'_2 = 3\mathbf{v}_2 = (-2, 1, 1)$$

$$\mathbf{v}_3 = \mathbf{u}_3 - \frac{\langle \mathbf{u}_3, \mathbf{v}_1 \rangle}{\|\mathbf{v}_1\|^2} \mathbf{v}_1 - \frac{\langle \mathbf{u}_3, \mathbf{v}'_2 \rangle}{\|\mathbf{v}'_2\|^2} \mathbf{v}'_2$$

$$= (0, 0, 1) - \frac{1}{3}(1, 1, 1) - \frac{1}{6}(-2, 1, 1)$$

$$= \left(-\frac{1}{3}, -\frac{1}{3}, \frac{2}{3} \right) + \left(\frac{2}{6}, -\frac{1}{6}, -\frac{1}{6} \right)$$

$$= \left(0, -\frac{1}{2}, \frac{1}{2} \right)$$

$$\mathbf{v}'_3 = 2\mathbf{v}_3 = (0, -1, 1)$$

$$\mathbf{q}_1 = \frac{1}{\|\mathbf{v}_1\|} \mathbf{v}_1 = \frac{1}{\sqrt{3}}(1, 1, 1)$$

$$\mathbf{q}_2 = \frac{1}{\|\mathbf{v}'_2\|} \mathbf{v}'_2 = \frac{1}{\sqrt{6}}(-2, 1, 1)$$

$$\mathbf{q}_3 = \frac{1}{\|\mathbf{v}'_3\|} \mathbf{v}'_3 = \frac{1}{\sqrt{2}}(0, -1, 1)$$