* Elementry row operations

(i) $aR_i + R_j$

(ii) aR_i

(iii) R_{ij}

* Jauss method to solve Linear system Equations

STEP1 organize the system.

STEP2 write the system as (Augmented matrix $[A:B]$

STEP3 use the elementry row operations to (eleminate) the augmented matrix

i.e. to get the form

$$\left[\begin{array}{ccc|c} 1 & \square & \square & a \\ 0 & 1 & \square & b \\ 0 & 0 & 1 & c \end{array}\right]$$

* How to Know the solution

we have 2 types of the system

(1) Homogenous

$AX = 0$

has only 2 Possibilities

- unique solution (Zero solution)
- ∞ many solutions

Number of veriables is bigger than number of Equations

$$\left[\begin{array}{ccc|c} 1 & \square & \square & a \\ 0 & 1 & \square & b \\ 0 & 0 & 0 & 0 \end{array}\right]$$

the solution will be written by Parametres

(number of Parameter = num of veriables − num of Equations)

(2) Non homogenous

$AX = B$

we have 3 possibilities

- unique solution

$$\left[\begin{array}{ccc|c} 1 & \square & \square & a \\ 0 & 1 & \square & b \\ 0 & 0 & 1 & c \end{array}\right]$$

- ∞ many solutions

$$\left[\begin{array}{ccc|c} 1 & \square & \square & a \\ 0 & 1 & \square & b \\ 0 & 0 & 0 & 0 \end{array}\right]$$

- No solution

$$\left[\begin{array}{ccc|c} 1 & \square & \square & a \\ 0 & 1 & \square & b \\ 0 & 0 & 0 & c \end{array}\right]$$

where $c \neq 0$

e.g : After elimination we get the following :

$$\left[\begin{array}{cccc|c} 1 & 2 & 3 & -1 & 5 \\ 0 & 6 & 1 & 0 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

The system has ∞ many solutions (i.e. the solution will be written by Parameters)

Equations : $x + 2y + 3z - f = 5 \rightsquigarrow (1)$

$6y + 1z = 3 \rightsquigarrow (2)$

Number of Parameters $= \boxed{4} - \boxed{2} = 2$

Let $\boxed{z = t}$

$\Rightarrow \; y = \dfrac{3-t}{6}$ (By Equation (2))

Let $\boxed{x = s}$ (By Equation (1))

$$f = x + 2y + 3z - 5$$
$$= s + \frac{3-t}{3} + 3t - 5$$

$$\therefore \; S = \left\{ \begin{bmatrix} s \\ \frac{3-t}{6} \\ t \\ s + \frac{3-t}{3} + 3t - 5 \end{bmatrix} ; \; s, t \in \mathbb{R} \right\}$$

e.g: After elemination we get the following

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

" It is clear that it is Homogenous "

$$\Rightarrow \left.\begin{array}{l} z = 0 \\ y + 3z = 0 \\ x + 2y - z = 0 \end{array}\right\} \Rightarrow \begin{array}{l} z = 0 \\ y = 0 \\ x = 0 \end{array} \Rightarrow S = \left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right\}$$

unique solution

e.g: After elemination we get the following:

$$\left[\begin{array}{cccc|c} 1 & 2 & 0 & -1 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

then, we have

$$\begin{array}{l} (1) \ldots\; y + z = 0 \\ (2) \ldots\; x + 2y - f = 0 \end{array} \Rightarrow$$

we have ∞ many solutions

(Nu of Para = Num of Ver - Num of Equations $= \boxed{4} - \boxed{2} = 2$)

Let $y = t \xrightarrow{\text{By }(1)} z = -t$

$x = s \xrightarrow{\text{By}(1)} f = x + 2y = s + 2t$

$$\therefore\; S = \left\{ \begin{bmatrix} s \\ t \\ -t \\ s+2t \end{bmatrix} ; \; s, t \in \mathbb{R} \right\}$$

e.g: Find value of a to make the system

① ~~unique~~ ∞ many solutions

② No Solution

③ unique solutio

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 0 & 1 & 5 & 2 \\ 0 & 0 & a-1 & 0 \end{array}\right]$$

Solution (1) If $a - 1 = 0$ then number of veriables $>$ Number of Equations then there are ∞ many solutions

$\Rightarrow$ if $\boxed{a = 1}$ then

(2) If $a - 1 \neq 0 \Rightarrow$ the system has unique solution

(3) It is (impossible ~~the case~~. the case of no solution)

Therefore, The system is Consistent