42) $y_g = y_h + y_p$

y_h : $y'' + y = 0$

$m^2 + 1 = 0$ (c.e)

$m^2 = -1 \Rightarrow m = i, m = -i$ (Complex)

$y_h = c_1 \cos x + c_2 \sin x$

$y_p = y_{p_1} + y_{p_2}$

y_{p_1} : $\cos(x)\, e^{0\cdot x}$

$0 + 1\cdot i = i$ is root of c.e

$y_{p_1} = x\,(A\cos x + B\sin x)\, e^{0\cdot x}$

y_{p_2} : $-\sin(2x)\, e^{0\cdot x}$

$0 + 2i = 2i$ is not root of c.e

$y_{p_2} = (\cos(2x) + 1)\sin(2x)\, e^{0\cdot x}$

$y_p = y_{p_1} + y_{p_2} = \ldots$ Continue

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43) $y'' - 2y' - 3y = 2\cos^2(x) = \cos(2x) + 1$

$y_g = y_h + y_p$

y_h : $y'' - 2y' - 3y = 0$

$m^2 - 2m - 3 = 0$ (c.e)

$(m-3)(m+1) = 0$

$m = 3, \; m = -1$

$y_h = c_1 e^{3x} + c_2 e^{-x}$

y_p : $y_{p_1} + y_{p_2}$

y_{p_1} : $\cos(2x)\, e^{0\cdot x}$

$0 + 2i = 2i$ is not root of (c.e)

$y_{p_1} = (A\cos(2x) + B\sin(2x))\, e^{0\cdot x}$

y_{p_2} : $1 = 1 \cdot e^{0\cdot x}$

0 is not root of c.e

$y_{p_2} = r \cdot e^{0\cdot x} = r$

$y_p = A\cos(2x) + B\sin(2x) + r$ (continue)

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(46) $y_g = y_h + y_p$

y_h : $y^{(4)} - y^{(3)} = 0$

$m^4 - m^3 = 0$ (c.e)

$m^3(m-1) = 0$

$m = 0$ (3 times)

$m = 1$

$y_h = c_1 e^{0\cdot x} + c_2 x e^{0\cdot x} + c_3 x^2 e^{0\cdot x} + c_5 e^{x}$

$= c_1 + c_2 x + c_3 x^2 + c_5 e^{x}$

$y_p = y_{p_1} + y_{p_2}$

y_{p_1} : $x\, e^{0\cdot x}$

0 is root 3 times of (c.e)

$y_{p_1} = x^3 (A_0 + A_1 x)\, e^{0\cdot x}$

y_{p_2} : e^{x}

1 is not root of (c.e)

$y_{p_2} = x \cdot A_2 e^{x}$

$y_p = \ldots + \ldots$ (continue)