25) $y_g = y_c + y_p$

y_c : $y''' + 4y' = 0$

$m^3 + 4m = 0$ (c.e)

$m(m^2+4) = 0 \to m = 0,\ m = 2i,\ m = -2i$

$y_c = c_1 e^{0x} + c_2 \cos(2x) + c_3 \sin(2x)$

y_p : $y_p = u_1 y_1 + u_2 y_2 + u_3 y_3$

$= u_1 \cdot 1 + u_2 \cos(2x) + u_3 \sin(2x)$ $\quad u_1 = ?$ $\quad u_2 = ?$ $\quad u_3 = ?$

$u_1' = \frac{W_1}{W}$, $u_2' = \frac{W_2}{W}$, $u_3' = \frac{W_3}{W}$

$$W = \begin{vmatrix} y_1 & y_2 & y_3 \\ y_1' & y_2' & y_3' \\ y_1'' & y_2'' & y_3'' \end{vmatrix} = \begin{vmatrix} 1 & \cos(2x) & \sin(2x) \\ 0 & -2\sin(2x) & 2\cos(2x) \\ 0 & -4\cos(2x) & -4\sin(2x) \end{vmatrix}$$

$= 8\sin^2(2x) + 8\cos^2(2x)$

$= 8[\sin^2(2x) + \cos^2(2x)]$

$= 8 \cdot 1 = 8$

$$W_1 = \begin{vmatrix} 0 & y_2 & y_3 \\ 0 & y_2' & y_3' \\ \sec(2x) & y_2'' & y_3'' \end{vmatrix} = \sec(2x) \cdot 2$$

$$W_2 = \begin{vmatrix} y_1 & 0 & y_3 \\ y_1' & 0 & y_3' \\ y_1'' & \sec(2x) & y_3'' \end{vmatrix} = -\sec(2x) \cdot 2\cos(2x) = -2$$

$$W_3 = \begin{vmatrix} y_1 & y_2 & 0 \\ y_1' & y_2' & 0 \\ y_1'' & y_2'' & \sec(2x) \end{vmatrix} = \sec(2x) \cdot (-2\sin 2x)$$

$u_1' = \frac{2\sec(2x)}{8} = \frac{1}{4}\sec(2x) \to u_1 = \int \frac{1}{4}\sec(2x)\,dx$

$= \frac{1}{8}\ln|\sec(2x) + \tan(2x)|$

$u_2' = -\frac{2}{8} = -\frac{1}{4} \to u_2 = \int -\frac{1}{4}\,dx = -\frac{1}{4}x$

$u_3' = \frac{-2\sin(2x)}{8\cos(2x)} \Rightarrow u_3 = \frac{1}{8}\int \frac{-2\sin(2x)}{\cos(2x)}\,dx$

$= \frac{1}{8}\ln|\cos(2x)|$

$y_p = \frac{1}{8}\ln|\sec(2x) + \tan(2x)| \cdot 1$

$+ \left(-\frac{1}{4}x\right)\cos(2x) + \frac{1}{8}\ln|\cos(2x)| \cdot \sin(2x)$

$y_g = \ldots\ldots + \ldots\ldots$