Exercises. In the following exercise answer true or false

1. The point $x_0 = -1$ is a regular singular point for the differential equation
$$(1-x^2)y'' - 2xy' + 12y = 0.$$

2. The point $x_0 = 0$ is an ordinary point for the differential equation
$$xy'' + (1-x)y' + 2y = 0.$$

Exercises

In exercises 1 through 9, locate the ordinary points, regular singular points and irregular singular points of the given differential equation

1) $xy'' - (2x+1)y' + y = 0.$
2) $(1-x)y'' - y' + xy = 0.$
3) $x^3(1-x^2)y'' + (2x-3)y' + xy = 0.$
4) $(1-x)^4y'' - xy = 0.$
5) $2x^2y'' + (x-x^2)y' - y = 0.$

In exercises 10 through 13 verify that all singular points of the differential equation are regular singular points

10) $x^2y'' + xy' + (x^2 - \nu^2)y = 0.$ (Bessel equation)
11) $(1-x^2)y'' - xy' + \nu^2 y = 0.$ (Chebyshev equation)
12) $xy'' + (1-x)y' + \nu y = 0.$ (Laguerre equation)
13) $(1-x^2)y'' - 2xy' + n(n+1)y = 0.$ (Legendre equation)

For the following equations, specify an interval around $x_0 = 0$ for which a power series solution converges

14) $y'' - xy' + 6y = 0.$
15) $(x^2-4)y'' - 2xy' + 9y = 0.$

In exercises 16 through 22 solve the initial value problems by using the method of power series about the given initial point x_0

16) $\begin{cases} (1-x^2)y'' - 2xy' + 6y = 0 \\ y(0) = 1,\ y'(0) = 0, \end{cases}$

20) $\begin{cases} y'' - 2(x-1)y' + 2y = 0 \\ y(1) = 0,\ y'(1) = 1, \end{cases}$

Solve the following equations in power series

29) $(3-x^2)y'' - 4xy' - 7y = 0.$
30) $(1-x^2)y'' - 3xy' + y = 0.$