

$$\textcircled{1} \begin{cases} Dx - x + y = -t^2 & (1) \\ -x + Dy - 3y = 2t & (2) \end{cases} \Rightarrow \begin{cases} x' + y - x = -t^2 \\ y' - x - 3y = 2t \end{cases}$$

$$\begin{cases} (D-1)x + y = -t^2 \\ -x + (D-3)y = 2t \end{cases} \xrightarrow{\text{جمع}} (D^2 - 4D + 4)y = -t^2 - 2t + 2$$

$$y = y_c + y_p$$

$$y_c: (D^2 - 4D + 4)y = 0$$

$$m^2 - 4m + 4 = 0$$

$$(m-2)(m-2) = 0$$

$$m_1 = 2 = m_2$$

$$y_c = c_1 e^{2t} + c_2 t e^{2t}$$

$$y_p = At^2 + Bt + C$$

$$y_p' = 2At + B$$

$$y_p'' = 2A$$

$$2A - 8A + 4B + 4A(2t) + 4Bt + 4C = -t^2 - 2t + 2$$

$$4A = -1 \Rightarrow A = -\frac{1}{4}$$

$$-8A + 4B = -2$$

$$2 + 4B = -2$$

$$4B = -4 \Rightarrow B = -1$$

$$2A - 4B + 4C = 2$$

$$-\frac{1}{2} + 4 + 4C = 2$$

$$4C = 2 - \frac{7}{2} = -\frac{3}{2}$$

$$C = -\frac{3}{8}$$

$$y_c = c_1 e^{2t} + c_2 t e^{2t} - \frac{1}{4} t^2 - t - \frac{3}{8}$$

$$\text{From eq 2:}$$

$$x + y' - 3y = 2t$$

$$= 2c_1 e^{2t} + 2c_2 t e^{2t} + 2c_2 t^2 e^{2t} - \frac{1}{2} t - 1 - 3c_1 e^{2t} - 3c_2 t e^{2t} + \frac{3}{4} t^2 + 3t + \frac{9}{8} - 2t$$

$$= (-c_1 + c_2) e^{2t} - c_2 t e^{2t} + \frac{3}{4} t^2 + \frac{1}{2} t + \frac{1}{8}$$

$$\textcircled{2} \begin{cases} D^2 y + Dx + x = 0 & (1) \\ Dy - y + x = \sin t & (2) \end{cases} \Rightarrow \begin{cases} y'' + x' + x = 0 \\ y' - y + x = \sin t \end{cases}$$

$$(D+1)x + D^2 y = 0 \quad (2)$$

$$x + (D-1)y = \sin t$$

$$\begin{cases} (D-1)x + D^2 y = 0 & -(D^2-1) \\ -(D+1)x - (D+1)(D-1)y = -(D^2-1)y \end{cases}$$

$$y = -\cos t - \sin t$$

From eq 2:

$$x = -y' + y + \sin t$$